

# AMAT 108: Elementary Statistics

Fall 2025

Final Exam – Version 1

December 10, 2025

✓	Section	Instructor Name	Meeting Time	Meeting Days	Classroom
	4863	John Habib	12:00PM	T/TH	SS 116
	4465	Tung Lam	8:00AM	M/W	SS 116
	4713		9:00AM	T/TH	LC 3B
	1712	James Lamatina	3:00PM	M/W	LC 2
	3749		10:30AM	T/TH	SS 116
	1711		12:00PM		LC 1
	1710	Chris Lange	3:00PM	T/TH	SS 116
	3370		4:30PM		
	5435	Doug Rosenberg	3:00PM	T/TH	TA 118
	3750	Sam Spellman	9:00AM	T/TH	SS 116
	3748	Alea Wittig	11:40AM	M/W	SS 116

Name: \_\_\_\_\_ **Version 1 Solutions** \_\_\_\_\_ Student ID: \_\_\_\_\_

**Directions:** You have **two hours** to answer the following questions. *No notes, textbooks, mobile phones or other aids are allowed. Only scientific calculators are allowed.* For all multiple-choice questions, select **one** answer from among the choices given. No explanation is required to be shown and no partial credit will be given. Make sure to **completely** fill in the circle corresponding to your chosen answer.

For all free-response questions, you **must** show all necessary work to receive full credit. An answer with no work, even if correct, will not receive full credit. Please circle or box your final answer. All work, if needed, is to be rounded to **at least five** decimal places.

Please choose your section with a checkmark (✓) in the left-most column of the table above.

*Do not detach any pages of this exam for any reason.*

**Exam Scoring:**

Questions	Possible Points	Points Earned
1-5	9	
6	8	
7	8	
8	5	
9	10	
10	8	
11	6	
12	8	
13	14	
14	13	
Subtotal	89	
Add: Optional Question 15		
Total Points	89	
Percentage		



6. The number of days in any week that it rains or snows in a major metropolitan area,  $X$ , has probability distribution given in the table below:

$X$	1	2	3
$P(X)$	0.32	0.53	0.15

- (a) Find the mean value of  $X$ . This gives the average number of days in any given week that it rains or snows in the major metropolitan area. Do *not* round your answer. (3 pts.)

**Solution:**

$$\mu = 1(0.32) + 2(0.53) + 3(0.15) \quad (1)$$

$$= 1.83 \quad (2)$$

- +1 for multiplying each possible value by associated probability
- +1 for adding products together
- +1 for (2)

**Note:**

- -2 if the student shows

$$\mu = \frac{1 + 2 + 3}{3} = 2 \quad (3)$$

- -3 if the student shows

$$\mu = \frac{0.32 + 0.53 + 0.15}{3} \approx 0.333 \quad (4)$$

- (b) Use your answer in (a) to find the standard deviation of  $X$ . Round your answer to *three* decimal places. (5 pts.)

**Solution:**

$$\sigma = \sqrt{0.32(1 - 1.83)^2 + 0.53(2 - 1.83)^2 + 0.15(3 - 1.83)^2} \quad (5)$$

$$\approx 0.664 \quad (6)$$

- +1 for showing all three squared deviations from mean
- +1 for multiplying each squared deviation by associated probability
- +1 for adding products together
- +1 for taking square root
- +1 for (6) with correct rounding

**Note:** Follow any mistake the student makes in part (a) above.

7. The time (in days) it takes for a package to be shipped from New York to Los Angeles,  $Y$ , has a uniform distribution on the interval  $(1, 12)$ .

(a) Find the height of the density curve. *Leave your answer in fractional form.* (2 pts.)

**Solution:**

$$f(x) = \frac{1}{12 - 1} = \frac{1}{11} \quad \longrightarrow \quad \text{height} = \frac{1}{11} \quad (7)$$

- +1 for  $12 - 1$  as work shown
- +1 for right-hand side of (7)

**Note:** Students need not find the density function to get full credit for this part.

(b) Compute the probability that a package, shipped from New York, takes at most five days to arrive in Los Angeles. *Hint:* Compute  $P(Y \leq 5)$ . (3 pts.)

**Solution:**

$$P(Y \leq 5) = \frac{5 - 1}{11} \quad (8)$$

$$= \frac{4}{11} \approx 0.36364 \quad (9)$$

- +2 for (8) (one point for numerator and one point for denominator)
- +1 for either answer given in (9)

**Note:**

- Follow any mistake the student makes in part (a) above.
- Students may give a decimal answer for (9) that is rounded to any number of decimal places, even though the solution key has it to five decimal places.

(c) Compute the probability that a package, shipped from New York, takes at least seven days to arrive in Los Angeles. *Hint:* Compute  $P(Y \geq 7)$ . (3 pts.)

**Solution:**

$$P(Y \geq 7) = \frac{12 - 7}{11} \quad (10)$$

$$= \frac{5}{11} \approx 0.45455 \quad (11)$$

- +2 for (10) (one point for numerator and one point for denominator)
- +1 for either answer given in (11)

**Note:** Both notes to the solution for part (b) also apply here.

8. Complete ONE of the following two questions. Both are examples of symmetric-limits problems. If you complete both, the first will be scored and the second will be ignored. Also assume all requirements for the distributions of  $\bar{X}$  and  $\hat{p}$  are met. (5 pts.)

**Option 1:** It is a widely-held belief that the lifespan of extra-large black ink cartridges, in number of printed pages, has a normal distribution with mean 284 and standard deviation 18.1. Compute the probability that a random sample of 100 extra-large black ink cartridges has an average lifespan between 280 pages and 288 pages. *Hint:* Compute  $P(280 \leq \bar{X} \leq 288)$ .

**Solution:**

$$P(280 \leq \bar{X} \leq 288) = P\left(\frac{280 - 284}{\left(\frac{18.1}{\sqrt{100}}\right)} \leq \frac{\bar{X} - 284}{\left(\frac{18.1}{\sqrt{100}}\right)} \leq \frac{288 - 284}{\left(\frac{18.1}{\sqrt{100}}\right)}\right) \quad (12)$$

$$\approx P(-2.21 \leq Z \leq 2.21) \quad (13)$$

$$= P(Z \leq 2.21) - P(Z \leq -2.21) \text{ OR } = 1 - 2P(Z \leq -2.21) \quad (14)$$

$$\approx 0.9864 - 0.0136 \text{ OR } \approx 1 - 2(0.0136) \quad (15)$$

$$= 0.9728 \quad (16)$$

- +1 each for (12), (13), (14), (15), and (16)

**Note:** Students may either use the usual method or use the properties of symmetric-limits problems to correctly answer this question. Pay careful attention to which method the student uses.

**Option 2:** It is a widely-held belief that 75% of all extra-large black ink cartridges fail to have enough ink to print a certain number of pages. Compute the probability that, from a random sample of 184 extra-large black ink cartridges, between 69% and 81% of the cartridges fail to have enough ink to print a certain number of pages. *Hint:* Compute  $P(0.69 \leq \hat{p} \leq 0.81)$ .

**Solution:**

$$P(0.69 \leq \hat{p} \leq 0.81) = P\left(\frac{0.69 - 0.75}{\sqrt{\frac{0.75(1-0.75)}{184}}} \leq \frac{\hat{p} - 0.75}{\sqrt{\frac{0.75(1-0.75)}{184}}} \leq \frac{0.81 - 0.75}{\sqrt{\frac{0.75(1-0.75)}{184}}}\right) \quad (17)$$

$$\approx P(-1.88 \leq Z \leq 1.88) \quad (18)$$

$$= P(Z \leq 1.88) - P(Z \leq -1.88) \text{ OR } = 1 - 2P(Z \leq -1.88) \quad (19)$$

$$\approx 0.9699 - 0.0301 \text{ OR } = 1 - 2(0.0301) \quad (20)$$

$$= 0.9398 \quad (21)$$

- +1 each for (17), (18), (19), (20), and (21)

**Note:** Students may either use the usual method or use the properties of symmetric-limits problems to correctly answer this question. Pay careful attention to which method the student uses.

9. Online college courses have become increasingly popular since the Covid pandemic. However, a university is proposing to eliminate these courses in favor of in-person, active learning for its students. From a survey of 846 students, 24% are in favor of the university's proposal.

- (a) The requirements for a central 97% confidence interval for the proportion,  $p$ , of all students in favor of the University's proposal are met. Find an appropriate critical number. (1 pt.)

**Solution:**

$$z^* \approx 2.170 \quad \boxed{+1} \quad (22)$$

- (b) Construct a central 97% confidence interval for the proportion of all students in favor of the university's proposal. Round *both* endpoints to *three* decimal places. (6 pts.)

**Solution:**

$$CI = 0.24 \pm 2.170 \sqrt{\frac{0.24(1-0.24)}{846}} = \left( 0.24 - 2.170 \sqrt{\frac{0.24(1-0.24)}{846}}, 0.24 + 2.170 \sqrt{\frac{0.24(1-0.24)}{846}} \right) \quad (23)$$

$$\approx (0.208, 0.272) \quad (24)$$

- $\boxed{+1}$  for fraction inside square root (no partial credit)
- $\boxed{+1}$  for taking square root of fraction
- $\boxed{+1}$  for multiplying 2.170 by square root of fraction
- $\boxed{+1}$  for adding and subtracting 0.24 from product
- $\boxed{+2}$  for (24) (one point for each endpoint)

**Note:**

- If the student shows all work and gives (0.272, 0.208) as their answer, deduct one point.
- Follow any mistake the student made in part (a) above.

- (c) Find the minimum sample size required to conservatively estimate  $p$  to within 0.008 with 97% confidence. Round your answer *up* to the nearest whole number. (3 pts.)

**Solution:**

$$n = \frac{1}{4} \left( \frac{2.170}{0.008} \right)^2 \quad (25)$$

$$\approx 18394.141 \quad (26)$$

$$\uparrow 18395 \quad (27)$$

- $\boxed{+1}$  for fraction inside of parentheses being squared (no partial credit)
- $\boxed{+1}$  for dividing squared fraction by 4
- $\boxed{+1}$  for (27)

10. A recent report claims that five-inch metal screws are not actually five inches in length. The authors of the report found that a sample of 98 five-inch screws had an average length of 5.3 inches with a standard deviation of 0.89 inches.

- (a) The requirements for a central 98% confidence interval for the mean,  $\mu$ , length of all five-inch screws are met. Find an appropriate critical number. (1 pt.)

**Solution:**

$$t^* \approx 2.365 \quad \boxed{+1} \quad (28)$$

- (b) Construct a central 98% confidence interval for the mean length of all five-inch screws. Round *both* endpoints to *three* decimal places. (6 pts.)

**Solution:**

$$CI = 5.3 \pm 2.365 \left( \frac{0.89}{\sqrt{98}} \right) \quad (29)$$

$$= \left( 5.3 - 2.365 \left( \frac{0.89}{\sqrt{98}} \right), 5.3 + 2.365 \left( \frac{0.89}{\sqrt{98}} \right) \right) \quad (30)$$

$$\approx (5.087, 5.513) \quad (31)$$

- $\boxed{+1}$  for numerator of fraction
- $\boxed{+1}$  for denominator of fraction including square root (no partial credit)
- $\boxed{+1}$  for multiplying 2.365 by fraction
- $\boxed{+1}$  for adding and subtracting 5.3 from product
- $\boxed{+2}$  for (31) (one point for each endpoint)

**Note:**

- If the student shows all work and gives (5.513, 5.087) as their answer, deduct one point.
- Follow any mistake the student made in part (a) above.

- (c) Which of the following is the correct interpretation of the interval constructed in (b) above? (1 pt.)

- ① We are 98% confident that the actual value of  $\mu$  is within the constructed interval.
- ② The probability that the actual value of  $\mu$  is within the constructed interval equals 98%.
- ③ Both of the previous answers are correct.
- ④ None of the previous options are true.



11. A recent report in a leading home improvement magazine claims that 63% of all homes with heating, ventilation, and air conditioning (HVAC) systems cost more than a certain threshold every month to operate. However, a research scientist found that 58% of 492 randomly-selected homes with HVAC systems cost more than the threshold every month to operate. Is there enough evidence to conclude that the report's claim is too high?

(a) Select the correct pair of statistical hypotheses. (1 pt.)

①  $H_0: p = 0.63$   
vs.  
 $H_1: p < 0.63$

②  $H_0: p = 0.63$   
vs.  
 $H_1: p > 0.63$

③  $H_0: p = 0.63$   
vs.  
 $H_1: p \neq 0.63$

- (b) The requirements for the One-Sample  $Z$  Test for  $p$  are found to be met. Compute the  $z^*$  test statistic. Round your answer to *two* decimal places. (3 pts.)

**Solution:**

$$z^* = \frac{0.58 - 0.63}{\sqrt{\frac{0.63(1-0.63)}{492}}} \quad (32)$$

$$\approx -2.30^* \quad (33)$$

- +1 for numerator in (32)
- +1 for denominator in (32) including square root (no partial credit)
- +1 for (33) with correct rounding

- (c) Compute the  $p$ -value of the test. (1 pt.)

**Solution:**

$$\alpha_0 = P_{H_0}(Z \leq -2.30) \approx 0.0107 \quad \boxed{+1} \quad (34)$$

- (d) Which of the following is the correct conclusion? (1 pt.)

- ① Fail to reject  $H_0$  at  $\alpha = 0.05$ . There is not enough evidence to conclude that the report's claim is too high.
- ② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . There is a slight amount of evidence to conclude that the report's claim is too high.
- ③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . There is a convincing amount of evidence to conclude that the report's claim is too high.
- ④ Reject  $H_0$  at  $\alpha = 0.001$ . There is an overwhelming amount of evidence to conclude that the report's claim is too high.

12. A recent report claims that standard D-cell batteries average a lifespan of 50 weeks. However, it is found that 105 randomly-selected standard D-cell batteries average a lifespan of 52.8 weeks with a standard deviation of 6.88 weeks. Is there enough evidence to conclude that the report's claim is too low?

(a) Select the correct pair of statistical hypotheses. (1 pt.)

①  $H_0: \mu = 50$   
vs.  
 $H_1: \mu < 50$

②  $H_0: \mu = 50$   
vs.  
 $H_1: \mu > 50$

③  $H_0: \mu = 50$   
vs.  
 $H_1: \mu \neq 50$

(b) The requirements for the One-Sample  $T$  Test for  $\mu$  are met. Compute the  $t^*$  test statistic. Round your answer to *one* decimal place. (3 pts.)

**Solution:**

$$t^* = \frac{52.8 - 50}{6.88/\sqrt{105}} \quad (35)$$

$$\approx 4.2^{***} \boxed{+1} \quad (36)$$

- $\boxed{+2}$  for (35) (one point for numerator and one point for entire denominator (no partial credit))

(c) The test statistic has a  $t$  distribution with how many degrees of freedom? (1 pt.)

① 107

② 104

③ 105

④ 106

(d) Compute the  $p$ -value of the test, and express your answer to *three* decimal places. (2 pts.)

**Solution:**

$$\alpha_0 = P_{H_0}(T \geq 4.2) \approx 0 \quad (37)$$

- $\boxed{+2}$  for (37) (one point for answer and one point for work)

(e) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject  $H_0$  at  $\alpha = 0.05$ . There is not enough evidence to conclude that the report's claim is too low.

② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . There is a slight amount of evidence to conclude that the report's claim is too low.

③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . There is a convincing amount of evidence to conclude that the report's claim is too low.

④ Reject  $H_0$  at  $\alpha = 0.001$ . There is an overwhelming amount of evidence to conclude that the report's claim is too low.

13. Having a cellphone is a must in today's world. Sometimes, though, people have preferences to type of cellphone. A survey was conducted to measure this, and the results are shown below:

Brand	A	B
Number Surveyed	500	600
Number Preferred	270	320
Preference Rate	54%	53.333%

Is there enough evidence to conclude that the proportion of all individuals preferring Brand A statistically differs from the proportion of those preferring Brand B? Use  $p_1$  to represent the proportion of all individuals preferring Brand A, and use  $p_2$  to represent the proportion of all individuals preferring Brand B.

- (a) Select the correct pair of statistical hypotheses. (1 pt.)

①  $H_0: p_1 = p_2$   
vs.  
 $H_1: p_1 < p_2$

②  $H_0: p_1 = p_2$   
vs.  
 $H_1: p_1 > p_2$

③  $H_0: p_1 = p_2$   
vs.  
 $H_1: p_1 \neq p_2$

- (b) Are all requirements of the Two-Sample  $Z$  Test for Equal Proportions met? (1 pt.)

① Yes, because both populations have normal distributions.

② Yes, because both sample sizes are sufficiently large.

③ Yes, because both sets of success-failure conditions are met.

④ No, because none of the above are true.

- (c) Using either method, compute the pooled sample proportion. Round your answer to *five* decimal places. (3 pts.)

**Solution:**

$$\hat{p}_c = \frac{500(0.54) + 600(0.53333)}{500 + 600}$$

$$\approx 0.53636$$

OR

$$\hat{p}_c = \frac{270 + 320}{500 + 600}$$

$$\approx 0.53636 \quad (39)$$

- +1 for numerator in (38)
- +1 for denominator in (38)
- +1 for (39) with correct rounding

**Note:** Either side is acceptable. Pay careful attention to which side the student uses.

(d) Compute the  $z^*$  test statistic. Round your answer to *two* decimal places. (5 pts.)

**Solution:**

$$z^* = \frac{0.54 - 0.53333}{\sqrt{\frac{0.53636(1-0.53636)}{500} + \frac{0.53636(1-0.53636)}{600}}} \quad (40)$$

$$\approx 0.22 \quad (41)$$

- +1 for numerator in (40)
- +2 for fractions in denominator of (40) (one point each)
- +1 for taking square root of sum of fractions
- +1 for (41) with correct rounding

(e) Compute the  $p$ -value of the test. (3 pts.)

**Solution:**

$$\alpha_0 = 2P_{H_0}(Z \leq -0.22) \quad (42)$$

$$\approx 2(0.4129) \quad (43)$$

$$= 0.8258 \quad (44)$$

- +1 for finding area of either tail to equal 0.4129, as in (43)
- +1 for doubling area of either tail, as in (42) and (43)
- +1 for (44)

(f) Which of the following is the correct conclusion? (1 pt.)

- ① Fail to reject  $H_0$  at  $\alpha = 0.05$ . There is not enough evidence to conclude that the proportion of individuals preferring Brand A statistically differs from the proportion of those preferring Brand B.
- ② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . There is a slight amount of evidence to conclude that the proportion of individuals preferring Brand A statistically differs from the proportion of those preferring Brand B.
- ③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . There is a convincing amount of evidence to conclude that the proportion of individuals preferring Brand A statistically differs from the proportion of those preferring Brand B.
- ④ Reject  $H_0$  at  $\alpha = 0.001$ . There is an overwhelming amount of evidence to conclude that the proportion of individuals preferring Brand A statistically differs from the proportion of those preferring Brand B.

14. In a 2019 paper published in the Proceedings of the National Academy of Sciences, Harvard University professors showed that promoting active learning in college courses improves student understanding. However, the paper doesn't clarify if this leads to better exam performance. To test this, randomly-selected students who didn't participate in the initial study were assigned to learn using standard or active learning practices. They then took the same exam, and one instructor graded all exams. Summary statistics are shown below:

Learning Method	Standard	Active
Exam Average	83.03	89.57
Standard Deviation	17.867	12.112
Number of Students	108	106

Is there enough evidence to conclude that there are significant differences in exam scores between the two groups of students?

- (a) Select the correct pair of statistical hypotheses. (1 pt.)

①  $H_0: \mu_1 = \mu_2$   
vs.  
 $H_1: \mu_1 < \mu_2$

②  $H_0: \mu_1 = \mu_2$   
vs.  
 $H_1: \mu_1 > \mu_2$

③  $H_0: \mu_1 = \mu_2$   
vs.  
 $H_1: \mu_1 \neq \mu_2$

- (b) Are all requirements for Welch's  $T$  Test met? (1 pt.)

① Yes, because both populations have normal distributions.

② Yes, because both samples have normal distributions.

③ Yes, because both samples are sufficiently large.

④ Yes, because the requirements for the One-Sample  $T$  Test for  $\mu$  are met for each sample, but none of the above are true.

⑤ No, because none of the above are true.

- (c) Compute  $df$  for the test. (2 pts.)

**Solution:**

$$df = \min\{108, 106\} - 1 \quad (45)$$

$$= 105 \quad (46)$$

- +1 for subtracting one from smaller of two sample sizes, as in (45)
- +1 for (46)

(d) Compute the  $t^*$  test statistic. Round your answer to *one* decimal place. (5 pts.)

**Solution:**

$$t^* = \frac{83.03 - 89.57}{\sqrt{\frac{17.867^2}{108} + \frac{12.112^2}{106}}} \quad (47)$$

$$\approx -3.1^{**} \quad (48)$$

- +1 for numerator in (47)
- +2 for fractions in denominator of (47) (one point each)
- +1 for taking square root of sum of fractions in denominator of (47)
- +1 for (48) with correct rounding

**Note:** If the student provides 3.1 as their answer, do not deduct any points.

(e) Compute the  $p$ -value of the test. (3 pts.)

**Solution:**

$$\alpha_0 = 2P_{H_0}(T \geq 3.1) \quad (49)$$

$$\approx 2(0.001) \quad (50)$$

$$= 0.002 \quad (51)$$

(f) Which of the following is the correct conclusion? (1 pt.)

- ① Fail to reject  $H_0$  at  $\alpha = 0.05$ . There is not enough evidence to conclude that significant differences in exam scores exist between the two groups of students.
- ② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . There is a slight amount of evidence to conclude that significant differences in exam scores exist between the two groups of students.
- ③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . There is a convincing amount of evidence to conclude that significant differences in exam scores exist between the two groups of students.
- ④ Reject  $H_0$  at  $\alpha = 0.001$ . There is an overwhelming amount of evidence to conclude that significant differences in exam scores exist between the two groups of students.

**This question is OPTIONAL and can only improve your exam score.**

15. As part of a study on how related exercise and total cholesterol levels are, seven UAlbany students were selected at random. The students' total cholesterol levels,  $Y$ , were measured, and they were asked how many minutes of vigorous exercise,  $X$ , they average in a day.

- (a) With  $\bar{x} \approx 51.4$ ,  $\bar{y} = 145$ ,  $s_x \approx 40.28$ ,  $s_y \approx 39$ , and  $n = 7$ , complete the table below to calculate  $r$ . Round *all* table entries to *three* decimal places, where appropriate.

$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
30	160	-21.4	15	-321
50	120	-1.4	-25	35
45	137	-6.4	-8	51.2
90	110	38.6	-35	-1351
15	185	-36.4	40	-1456
10	203	-41.4	58	-2401.2
120	100	68.6	-45	-3087
Sum =				-8530
$(n - 1) \times s_x \times s_y \approx$				9425.52
$r \approx$				-0.905

- (b) Find the least-squares regression line for the data. Round *both* coefficients to *three* decimal places.

**Solution:**

$$b = \frac{(-0.905)(39)}{40.28} \approx -0.876 \quad a = 145 - (-0.876)(51.4) \approx 190.026 \quad \hat{Y} = 190.026 - 0.876X \quad (52)$$

### Formula Sheet:

- Probability:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) \text{ if events } A \text{ and } B \text{ are disjoint}$$

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B) \text{ if events } A \text{ and } B \text{ are independent}$$

- Complement probability (probability complement rule):  $P(A^C) = 1 - P(A)$

- Conditional probability of event  $A$  given event  $B$  occurs:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- At least 1 Rule:  $P(\text{at least 1 success in } n \text{ trials}) = 1 - P(\text{no successes in } n \text{ trials})$

- Finding the height of a uniform distribution:

$$\text{Height} = \frac{1}{\text{Base}} = \frac{1}{b - a}$$

- Probability (area) of a uniform distribution: Probability (or area) = Base  $\cdot$  Height

- Mean of discrete random variable  $X$  with possible values  $x_1, x_2, \dots, x_n$ :

$$\mu_X = E(X) = x_1p(x_1) + x_2p(x_2) + \dots + x_np(x_n)$$

- Standard deviation of a discrete random variable  $X$  with possible values  $x_1, x_2, \dots, x_n$ :

$$\sigma_X = \sqrt{(x_1 - \mu_X)^2p(x_1) + (x_2 - \mu_X)^2p(x_2) + \dots + (x_n - \mu_X)^2p(x_n)}$$

or

$$\sigma_X = \sqrt{E(X^2) - (E(X))^2}$$

- Mean and standard deviation for sampling distribution of  $\bar{X}$ :

$$\mu_{\bar{X}} = \mu \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

where  $\mu$  = population mean and  $\sigma$  = population standard deviation



- Standardized variable ( $z$ -score) for  $\bar{X}$  (when  $\sigma$  known):

$$Z = \frac{\bar{X} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

- Mean and standard deviation for sampling distribution of  $\hat{p}$ :

$$\mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

where  $p$  = population proportion of successes

- Sample proportion of successes:

$$\hat{p} = \frac{\text{number of successes in the sample}}{n}$$

- Standardized variable ( $z$ -score) for  $\hat{p}$ :

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- Success/failure condition for the sampling distribution of  $\hat{p}$  to be approximately normal:

$$np \geq 10 \qquad \text{and} \qquad n(1-p) \geq 10$$

- Confidence interval for a population proportion:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Determining the sample size needed to be within  $M$  = margin of error under a certain confidence level:

$$n = p(1-p) \left(\frac{z^*}{M}\right)^2$$

- To find the conservatively large sample size needed, set  $p = 0.5$ .

- Confidence interval for a population mean (when  $\sigma$  unknown):

$$\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}}\right) \qquad df = n - 1$$

- $z^*$  test statistic for hypothesis testing of a population proportion:

$$z^* = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- $t^*$  test statistic for hypothesis testing of a population mean (when  $\sigma$  unknown):

$$t^* = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} \quad df = n - 1$$

- Hypothesis testing for the difference of two means (Welch's  $T$  Test):

$$t^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\ df = \text{smaller of } n_1 - 1 \text{ and } n_2 - 1$$

- Hypothesis testing for the difference of two proportions (2-Sample  $Z$  Test for Equal Proportions):

$$z^* = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}}$$

- Combined/pooled proportion:

$$\hat{p}_c = \frac{n_1 \cdot \hat{p}_1 + n_2 \cdot \hat{p}_2}{n_1 + n_2} \quad \text{or} \quad \hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

- Success/failure condition for the 2-Sample  $Z$  Test for Equal Proportions:

$$n_1 \hat{p}_1 \geq 10 \quad n_1(1 - \hat{p}_1) \geq 10 \quad n_2 \hat{p}_2 \geq 10 \quad n_2(1 - \hat{p}_2) \geq 10$$

- Correlation coefficient:

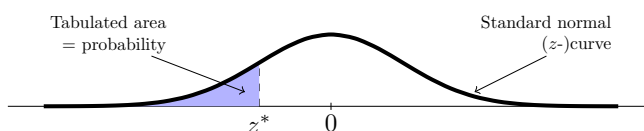
$$r = \frac{(x_1 - \bar{x})(y_1 - \bar{y}) + (x_2 - \bar{x})(y_2 - \bar{y}) + \cdots + (x_n - \bar{x})(y_n - \bar{y})}{(n-1) \cdot s_x \cdot s_y}$$

- Least-squares regression line:

$$\hat{y} = a + b \times x \quad b = r \cdot \frac{s_y}{s_x} \quad a = \bar{y} - b \cdot \bar{x}$$

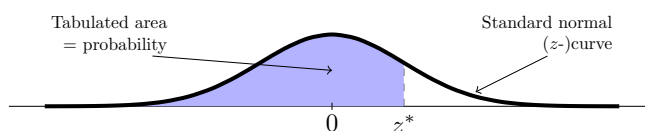
- Residual =  $y - \hat{y}$

***Standard Normal Table***



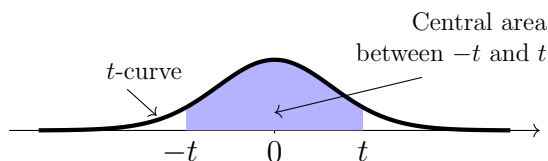
$z^*$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

*Standard Normal Table*



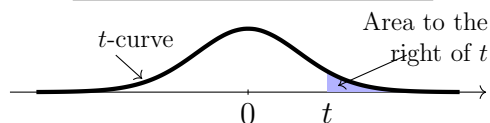
$z^*$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

*t-Distribution Table of Critical Values*



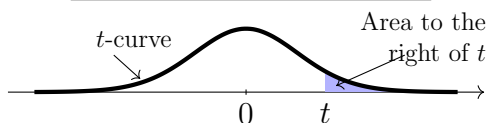
degree of freedom	Central Area Captured / Confidence Level								
	80%	90%	95%	96%	97%	98%	99%	99.8%	99.9%
1	3.078	6.314	12.706	15.895	21.205	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	4.849	5.643	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	3.482	3.896	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	2.999	3.298	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	2.757	3.003	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	2.612	2.829	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.517	2.715	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.449	2.634	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.398	2.574	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.359	2.527	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.328	2.491	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.303	2.461	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.282	2.436	2.650	3.012	3.852	4.221
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
96	1.290	1.661	1.985	2.082	2.203	2.366	2.628	3.177	3.395
97	1.290	1.661	1.985	2.082	2.202	2.365	2.627	3.176	3.394
98	1.290	1.661	1.984	2.081	2.202	2.365	2.627	3.175	3.393
99	1.290	1.660	1.984	2.081	2.202	2.365	2.626	3.175	3.392
100	1.290	1.660	1.984	2.081	2.201	2.364	2.626	3.174	3.390
101	1.290	1.660	1.984	2.081	2.201	2.364	2.625	3.173	3.389
102	1.290	1.660	1.983	2.080	2.201	2.363	2.625	3.172	3.388
103	1.290	1.660	1.983	2.080	2.201	2.363	2.624	3.171	3.388
104	1.290	1.660	1.983	2.080	2.200	2.363	2.624	3.170	3.387
105	1.290	1.659	1.983	2.080	2.200	2.362	2.623	3.170	3.386
106	1.290	1.659	1.983	2.079	2.200	2.362	2.623	3.169	3.385
107	1.290	1.659	1.982	2.079	2.199	2.362	2.623	3.168	3.384
108	1.289	1.659	1.982	2.079	2.199	2.361	2.622	3.167	3.383
109	1.289	1.659	1.982	2.079	2.199	2.361	2.622	3.167	3.382
110	1.289	1.659	1.982	2.078	2.199	2.361	2.621	3.166	3.381
111	1.289	1.659	1.982	2.078	2.198	2.360	2.621	3.165	3.380
112	1.289	1.659	1.981	2.078	2.198	2.360	2.620	3.165	3.380
113	1.289	1.658	1.981	2.078	2.198	2.360	2.620	3.164	3.379
114	1.289	1.658	1.981	2.078	2.198	2.360	2.620	3.163	3.378
115	1.289	1.658	1.981	2.077	2.197	2.359	2.619	3.163	3.377
116	1.289	1.658	1.981	2.077	2.197	2.359	2.619	3.162	3.376
117	1.289	1.658	1.980	2.077	2.197	2.359	2.619	3.161	3.376
118	1.289	1.658	1.980	2.077	2.197	2.358	2.618	3.161	3.375
119	1.289	1.658	1.980	2.077	2.196	2.358	2.618	3.160	3.374
120	1.289	1.658	1.980	2.076	2.196	2.358	2.617	3.160	3.373
$z$ -critical = $\infty$	1.282	1.645	1.960	2.054	2.170	2.326	2.576	3.090	3.291

### Tail Areas for $t$ Curves



$t^* \backslash df$	1	2	...	94	95	96	97	98	99	100
0.0	0.500	0.500	...	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.468	0.465	...	0.460	0.460	0.460	0.460	0.460	0.460	0.460
0.2	0.437	0.430	...	0.421	0.421	0.421	0.421	0.421	0.421	0.421
0.3	0.407	0.396	...	0.382	0.382	0.382	0.382	0.382	0.382	0.382
0.4	0.379	0.364	...	0.345	0.345	0.345	0.345	0.345	0.345	0.345
0.5	0.352	0.333	...	0.309	0.309	0.309	0.309	0.309	0.309	0.309
0.6	0.328	0.305	...	0.275	0.275	0.275	0.275	0.275	0.275	0.275
0.7	0.306	0.278	...	0.243	0.243	0.243	0.243	0.243	0.243	0.243
0.8	0.285	0.254	...	0.213	0.213	0.213	0.213	0.213	0.213	0.213
0.9	0.267	0.232	...	0.185	0.185	0.185	0.185	0.185	0.185	0.185
1.0	0.250	0.211	...	0.160	0.160	0.160	0.160	0.160	0.160	0.160
1.1	0.235	0.193	...	0.137	0.137	0.137	0.137	0.137	0.137	0.137
1.2	0.221	0.177	...	0.117	0.117	0.117	0.117	0.117	0.117	0.116
1.3	0.209	0.162	...	0.098	0.098	0.098	0.098	0.098	0.098	0.098
1.4	0.197	0.148	...	0.082	0.082	0.082	0.082	0.082	0.082	0.082
1.5	0.187	0.136	...	0.068	0.068	0.068	0.068	0.068	0.068	0.068
1.6	0.178	0.125	...	0.056	0.056	0.056	0.056	0.056	0.056	0.056
1.7	0.169	0.116	...	0.046	0.046	0.046	0.046	0.046	0.046	0.046
1.8	0.161	0.107	...	0.038	0.038	0.038	0.037	0.037	0.037	0.037
1.9	0.154	0.099	...	0.030	0.030	0.030	0.030	0.030	0.030	0.030
2.0	0.148	0.092	...	0.024	0.024	0.024	0.024	0.024	0.024	0.024
2.1	0.141	0.085	...	0.019	0.019	0.019	0.019	0.019	0.019	0.019
2.2	0.136	0.079	...	0.015	0.015	0.015	0.015	0.015	0.015	0.015
2.3	0.131	0.074	...	0.012	0.012	0.012	0.012	0.012	0.012	0.012
2.4	0.126	0.069	...	0.009	0.009	0.009	0.009	0.009	0.009	0.009
2.5	0.121	0.065	...	0.007	0.007	0.007	0.007	0.007	0.007	0.007
2.6	0.117	0.061	...	0.005	0.005	0.005	0.005	0.005	0.005	0.005
2.7	0.113	0.057	...	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.8	0.109	0.054	...	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.9	0.106	0.051	...	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.0	0.102	0.048	...	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.1	0.099	0.045	...	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.096	0.043	...	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3	0.094	0.040	...	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.4	0.091	0.038	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.5	0.089	0.036	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.6	0.086	0.035	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.7	0.084	0.033	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.8	0.082	0.031	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.9	0.080	0.030	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.078	0.029	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.1	0.076	0.027	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.2	0.074	0.026	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.3	0.073	0.025	...	0.000	0.000	0.000	0.000	0.000	0.000	0.000

### Tail Areas for $t$ Curves



$t^* \backslash df$	101	102	103	104	105	106	107	108	109	110
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460
0.2	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421
0.3	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382	0.382
0.4	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.345
0.5	0.309	0.309	0.309	0.309	0.309	0.309	0.309	0.309	0.309	0.309
0.6	0.275	0.275	0.275	0.275	0.275	0.275	0.275	0.275	0.275	0.275
0.7	0.243	0.243	0.243	0.243	0.243	0.243	0.243	0.243	0.243	0.243
0.8	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213
0.9	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
1.0	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
1.1	0.137	0.137	0.137	0.137	0.137	0.137	0.137	0.137	0.137	0.137
1.2	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116
1.3	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098
1.4	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082
1.5	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068
1.6	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056
1.7	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
1.8	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
1.9	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
2.0	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
2.1	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019
2.2	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
2.3	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012
2.4	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
2.5	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
2.6	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
2.7	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.8	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000