University at Albany
State University of New York

AMAT113 Calculus II — Final Exam B — Fall 2024

Print Name:

UAlbany Email:

Please indicate your lecture section with a check mark (✓) in the leftmost column.

✓	Class No	Professor	Time and location
	3891	Dr. Muller	MWF 1:10-2:25PM HU0132
	3892	Dr. Muller	MWF 11:40-12:55PM HU0128
	3893	Dr. Beckhardt	MWF 11:40-12:55PM HU0133
	3895	Dr. Beckhardt	MWF 8:00-9:15AM HU0123
	3894	Dr. Wittig	MWF 3:00-4:15PM HU0133
	4779	Mr. Gelnett	MWF 1:10-2:25PM HU0123

Directions: You have **120 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible. An answer with no work, even if correct, will not receive full credit. Clearly circle or box your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points	Problem	Possible	Points
1	12		6	10	
2	10		7	10	
3	10		8	10	
4	8		9	10	
5	10		10	10	
			11**	8	
Total (Out of 100) =					

**Optional Extra Credit Problem

(1) Let $\mathcal{R}$ be the region bounded by the curve $y = x^2 + 1$ and the line $y = 2x + 4$.

(a) (6 Points) **Set up, but do not evaluate,** the integral of the area of the region $\mathcal{R}$.

(b) (6 Points) **Set up, but do not evaluate,** the integral of the volume of the solid obtained by rotating the region $\mathcal{R}$ about the $x-$axis.

(2) (a) (7 Points) Evaluate the following indefinite integral:

$$\int x^3 \ln(3x)\, dx.$$

(b) (3 Points) Consider the answer you obtained in Part (a). Determine whether or not your answer is correct. Provide a clear explanation of your reasoning. (*Hint: What is the relationship between differentiation and integration?*)

(3) (a) (4 Points) Compute the following derivative:

$$\frac{d}{dx}\left(\int_{e^x}^{5} \cos(t^2)\,dt\right).$$

(b) (6 Points) Evaluate the following indefinite integral:

$$\int 9x^2\sqrt{x^3+1}\,dx.$$

(4) (8 Points) Compute the definite integral:

$$\int_0^1 \frac{1}{(x+1)(x+3)}\, dx.$$

(5) (5 Points Each) Determine whether the following improper integrals converge or diverge. If the integral converges, then compute its value. If the integral diverges, then use the Comparison Test to justify its divergence.

(a) $\displaystyle\int_1^{\infty} \frac{e^x}{1+x}\,dx$

(b) $\displaystyle\int_0^{\infty} \frac{1}{9+x^2}\,dx$

(6) (5 Points Each) Determine whether each of the following series converge or diverge. If the series converges, then find its sum. If the series diverges, then justify why it diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

(7) (5 Points Each) Determine whether each of the following series converge or diverge.

Clearly justify your answer by indicating what test you used and explaining your reasoning. Show your work to receive full credit.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{2n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^{3/2}}$

(8) (10 Points) Find the interval of convergence of the series:

$$\sum_{n=0}^{\infty} \frac{(2x-1)^n}{2^n+1}.$$

Make sure to check the endpoints.

(9) (10 Points) Find the Taylor polynomial of degree 3 near the point $a = 1$ of the function $f(x) = \ln(x^2)$.

(10) (10 Points) Find the power series of the function $f(x)$ centered at $a = 0$ and state its radius of convergence.

$$f(x) = \frac{x}{1 - x^2}$$

Optional: Extra Credit Problem (8 Points)

(11) (a) (3 Points) Consider the parametric equations: $x(t) = -2t^2$, $y(t) = -1 + 2t$. Eliminate the parameter t to obtain a Cartesian equation (in terms of x and y only).

(b) (2 Points) Convert the polar coordinates $\left(9, \frac{\pi}{3}\right)$ to Cartesian coordinates.

(c) (3 Points) Rewrite the polar equation $r = 2\sin(\theta)$ as a Cartesian equation (in terms of x and y only).

Finite Geometric Sum:

$$a + ax + ax^2 + \ldots + ax^{n-1} = \frac{a(1-x^n)}{1-x}$$

Infinite Geometric Series: $a + ax + ax^2 + \ldots = \dfrac{a}{1-x}$ for $|x| < 1$

Ratio Test: For the series $\sum a_n$, suppose, $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = L$

If $L < 1$, then the series converges.

If $L > 1$, then the series diverges.

If $L = 1$, then the test is inconclusive.

Taylor Series of $f(x)$ centered at $x = a$:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

Taylor Series of important functions:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - \ldots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \quad \text{for } -1 < x < 1$$

Arc Length of the curve $y = f(x)$ on the interval $a \le x \le b$:

$$\int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Integration by Parts:

$$\int u dv = uv - \int v du \qquad \text{or} \qquad \int uv' dx = uv - \int vu' dx$$

Useful Integrals for Comparison:

$\displaystyle\int_1^\infty \frac{1}{x^p}\, dx$ converges for $p > 1$ and diverges for $p \le 1$.

$\displaystyle\int_0^1 \frac{1}{x^p}\, dx$ converges for $p < 1$ and diverges for $p \ge 1$.

$\displaystyle\int_0^\infty e^{-ax}\, dx$ converges for $a > 0$.

Differentiation formulas

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n \, dx = \frac{1}{n+1}x^{n+1} + C, \ (n \neq -1)$
2. $\int \frac{1}{x} \, dx = \ln\lvert x\rvert + C$
3. $\int a^x \, dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x \, dx = x\ln x - x + C$
5. $\int \sin ax \, dx = -\frac{1}{a}\cos ax + C$
6. $\int \cos ax \, dx = \frac{1}{a}\sin ax + C$
7. $\int \tan ax \, dx = -\frac{1}{a}\ln\lvert\cos ax\rvert + C$
8. $\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C, \ a \neq 0$