UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT113 Calculus II   Exam 3A   Spring 2025

Print Name:

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Instructors Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	12	
2	12	
3	12	
4	10	
5	10	
Total	56	

(Similar to Practice Assessment 13 and HW10)

(1) (a) Use the Comparison Test to determine whether the given infinite series converges or diverges. *Clearly justify your answer.*

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2+3}$$

(b) Use the Limit Comparison Test to determine whether the given infinite series converges or diverges. *Clearly justify your answer.*

$$\sum_{k=1}^{\infty} \frac{5k^2+1}{2k^3-k}$$

(Similar to Practice Assessment 14, 15 and HW10) (6 Points Each)

(2) (a) Use the Ratio Test to determine whether the given series converges or diverges. *Clearly justify your answer.*

$$\sum_{k=1}^{\infty} \frac{3k^2}{(k-1)!}$$

(b) Determine whether the series converges absolutely, converges conditionally, or diverges. *Clearly justify your answer.*

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{5k-1}$$

(Similar to Practice Assessment 16 and HW11) (6 Points Each)

(3) Consider the power series:

$$\sum_{k=0}^{\infty} \frac{k(x-3)^k}{2^k}.$$

The radius of convergence is ________________.

The interval of convergence is ________________.

Don't forget to check the endpoints!

(Similar to Practice Assessment 17 and HW11)

(4) (a) (6 Points) Find the power series representation for the given function by differentiating a known power series.

$$f(x) = \frac{3}{(1+x)^2}$$

Hint: $f(x) = -3\frac{d}{dx}\left(\frac{1}{1+x}\right)$.

(b) (4 Points) Find the power series representation for the given function by integrating a known power series.

$$g(x) = \ln(1-x)$$

Hint: $g(x) = \int \frac{-1}{1-x} dx$.

(Similar to Practice Assessment 18 and HW11)

(5) (a) (6 Points) Find the Taylor series for the function $f(x) = xe^{-2x}$ expanded about $x = 0$.

(b) (4 Points) Find the 4^{th} degree Taylor Polynomial for the function $f(x) = xe^{-2x}$ expanded about $x = 0$.

The Comparison Test: Let $\sum a_k$ and $\sum b_k$ be infinite series with $0 \leq a_k \leq b_k$ for each $k = 1, 2, \ldots$. Then

(i) If $\sum b_k$ converges, then so does $\sum a_k$.
(ii) If $\sum a_k$ diverges, then so does $\sum b_k$.

The Limit Comparison Test: Let $\sum a_k$ and $\sum b_k$ be infinite series with $a_k \geq 0$, $b_k > 0$ for all $k = 1, 2, \ldots$. If the limit

$$\rho = \lim_{k\to\infty} \frac{a_k}{b_k}$$

exists and $\rho \neq 0$, then either both series converge or both series diverge.

Alternating Series Test: Let $a_k \geq 0$ for each $k = 0, 1, 2, 3, \ldots$. The alternating series

$$\sum_{k=0}^{\infty} (-1)^k a_k$$

converges if both the following hold:

(i) The sequence $\{a_k\}_{k=0}^{\infty}$ is decreasing, and
(ii) $\lim_{k\to\infty} a_k = 0$.

The Ratio Test: Let $a_k \geq 0$ for all $k = 1, 2, \ldots$ and let

$$\rho = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}.$$

Then, provided the limit exits,

(i) If $\rho < 1$, the series $\sum_{k=1}^{\infty} a_k$ converges.
(ii) If $\rho > 1$, the series $\sum_{k=1}^{\infty} a_k$ diverges.
(iii) If $\rho = 1$, no conclusion may be drawn.

The Root Test: Let $a_k \geq 0$ for all $k = 1, 2, \ldots$ and let

$$\rho = \lim_{k\to\infty} \sqrt[k]{a_k}.$$

Then, provided the limit exits,

(i) If $\rho < 1$, the series $\sum_{k=1}^{\infty} a_k$ converges.
(ii) If $\rho > 1$, the series $\sum_{k=1}^{\infty} a_k$ diverges.
(iii) If $\rho = 1$, no conclusion may be drawn.

Absolute Convergence: The series $\sum a_k$ is said to **converge absolutely** if the series of absolute values, $\sum |a_k|$, converges.

Conditional Convergence: The series $\sum a_k$ **converges conditionally** if $\sum a_k$ converges and $\sum |a_k|$ diverges.

Taylor and Maclaurin Series: If f has derivatives of all orders at $x = a$, then the **Taylor Series** for the function f at a is

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots.$$

The Taylor series for f at $a = 0$ is known as the **Maclaurin** series for f.

Taylor Polynomial: If f has n derivatives at $x = a$, then the n-th **Taylor Polynomial** for the function f at $x = a$ is

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f(n)(a)}{n!}(x-a)^n.$$

The n-th Taylor polynomial for f at $a = 0$ is known as the n-th **Maclaurin Polynomial** for f.

Geometric Series: $\displaystyle\sum_{n=0}^{\infty} ax^n = a + ax + ax^2 + ax^3 + \ldots = \frac{a}{1-x} \quad \text{for} \quad |x| < 1.$

p-Series: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for all $p > 1$.

Taylor Series of important functions:

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - \ldots$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \ldots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \quad \text{for } -1 < x < 1$$