UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT113 Calculus II — Exam 2A — Spring 2025

Print Name:

UAlbany Email:

Instructors Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	8	
2	10	
3	8	
4	8	
5	8	
6	8	
7**	5	
Total	50	

**Optional Extra Credit Problem

(Similar to Practice Assessment 6 and HW6 Question 3)

(1) (8 points) Use integration by parts to evaluate the following integral.

$$\int_{1}^{e} 3t^2 \ln(t)\, dt$$

(Similar to Practice Assessment 7 and HW7 Questions 1-2)

(2) (5 points each) Evaluate the following integrals using known trigonometric methods or substitution.

(a) $\displaystyle\int \cos^5(x)\sin(x)\,dx$

(b) $\displaystyle\int \frac{4}{\sqrt{x^2-36}}\,dx$

(Similar to Practice Assessment 8 and HW8 Questions 1-5)

(3) (a) (4 Points) The partial fraction decomposition of $\dfrac{x^2}{x^2-36}$ can be written in the form

$$1+\frac{A}{x+6}+\frac{B}{x-6}.$$

The value of $A =$____________ and $B =$____________.

(b) (4 Points) Integrate $\displaystyle\int \frac{x^2}{x^2-36}\,dx$.

(Similar to Practice Assessment 9 and HW8 Questions 6-11)

(4) (4 points each) Determine whether the given improper integrals converges or diverges. If it converges, compute its value. If it diverges, justify why.

(a) $\displaystyle\int_{-\infty}^{0} \frac{2x}{x^2+15}\,dx$

(b) $\displaystyle\int_{-4}^{5} \frac{1}{2\sqrt{x+4}}\,dx$

(Similar to Practice Assessment 10 and HW9 Questions 3,4)

(5) (4 points each) Determine whether the following sequences converge or diverge. If the sequence converges, then compute its limit. If it diverges, justify why.

(a) $\left\{ \dfrac{n^3 + 5n^2 - 12}{4n^3 - 2n^2 + 2n + 1} \right\}$

(b) $\left\{ (-1)^{n+1} \left(\dfrac{n-1}{3n^2} \right) \right\}$

(Similar to Practice Assessment 11 and HW9 Questions 5-11)

(6) (4 points each) Determine whether the given infinite series converges or diverges. If it converges, find its sum. If it diverges, justify why.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{2^k}{5^k}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k^2+2}$

(Similar to Practice Assessment 12 and HW9 Questions 12-15)

(7) **(Optional Extra Credit Problem: 5 Points)** Use the Integral Test to justify whether the given infinite series converges or diverges. *You must use the Integral Test to receive credit.*

$$\sum_{k=1}^{\infty} \frac{1}{3k+2}$$

Integration by Parts:

$$\int u dv = uv - \int v du \qquad \text{or} \qquad \int uv' dx = uv - \int vu' dx$$

Trigonometric Identities and Substitutions:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \tan^2(\theta) + 1 = \sec^2(\theta)$$

Useful trigonometric substitutions used to handle integrals involving radicals:

$$\sqrt{a^2 + x^2} \quad \text{requires} \quad x = a\tan\theta$$

$$\sqrt{a^2 - x^2} \quad \text{requires} \quad x = a\sin\theta$$

$$\sqrt{x^2 - a^2} \quad \text{requires} \quad x = a\sec\theta$$

Infinite Geometric Series: $\displaystyle\sum_{n=0}^{\infty} ax^n = a + ax + ax^2 + \ldots = \frac{a}{1-x}$ for $|x| < 1$

Differentiation formulas

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln\|x\|) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\sec(x)) = \sec x\tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x\cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\, dx = \frac{1}{n+1}x^{n+1} + C,\ (n \neq -1)$
2. $\int \frac{1}{x}\, dx = \ln|x| + C$
3. $\int a^x\, dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\, dx = x\ln x - x + C$
5. $\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
6. $\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
7. $\int \tan ax\, dx = -\frac{1}{a}\ln|\cos ax| + C$
8. $\int \sec x\, dx = \ln|\sec x + \tan x| + C$