UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT113 Calculus II Exam 1A Spring 2025

Print Name:

UAlbany Email:

Instructors Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	8	
2	12	
3	12	
4	10	
5	7	
6	10	
7**	6	
Total	59	

**Optional Extra Credit Problem

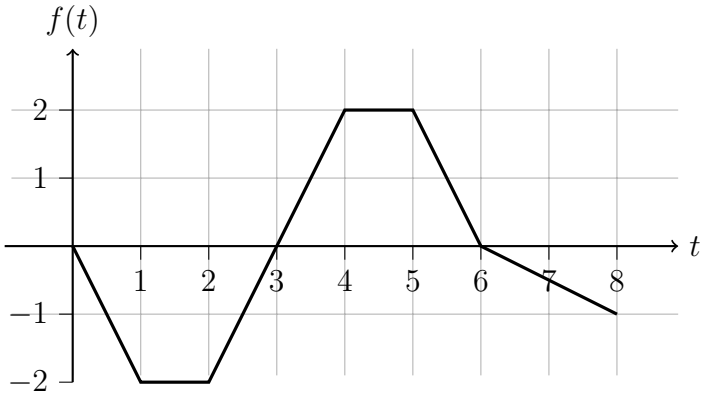
(1) A table of values of a function f is given below.

x	0	9	18	27	36
$f(x)$	10	-15	25	-5	-10

(a) (4 Points) Find L_4, a left-endpoint approximation with $n = 4$ for $\int_0^{36} f(x)dx$.

(b) (4 Points) Find R_4, a right-endpoint approximation with $n = 4$ for $\int_0^{36} f(x)dx$.

(2) (3 Points Each) Let $g(x) = \int_0^x f(t)dt$ where f is defined on the interval $[0, 8]$ and whose graph is shown below.



Fill in the blanks.

(a) $g(3) =$ _____.

(b) $g(6) =$ _____.

(c) $g'(4) =$ _____.

(d) The critical points of g are _____.

(3) (a) (6 Points) Use geometry and properties of integrals to evaluate the integral:

$$\int_0^4 \left(|2-x|+5+\sqrt{16-x^2}\right) dx$$

(b) (6 Points) Compute the derivative:

$$\frac{d}{dx}\left(\int_{-3}^{\sin x} \frac{\sqrt{t^2-1}}{t} dt\right)$$

(4) (5 Points Each) Evaluate each integral below.

(a) $\int \frac{x}{(5x^2+14)^7} dx$

(b) $\int_0^{\pi/6} e^{2\sin(t)} \cos(t) dt$

(c) $\int xe^{3x} dx$

(Optional Extra Credit Problem)

(5) (a) (3 Points) Sketch the region bounded by the curves $y = e^{3x}$, $y = e^{6x}$, and $x = 1$.

(b) (4 Points) Find the area of the region sketched in Part (a).

(6) (a) (5 Points) Consider the region bounded by the curves $y = \sqrt{4 - x^2}, y = 1$, and $x = 0$. Set up, but do not evaluate, an integral to find the volume of the solid obtained by rotating the region around the x-axis.

(b) (5 Points) Consider the region bounded by the curves $y = \sqrt{4 - x^2}, y = 1$, and $x = 0$. Set up, but do not evaluate, an integral to find the volume of the solid obtained by rotating the region around the y-axis.

(7) *(Optional Extra Credit)* The base of a solid is the region bounded by the curves $y = x^2$ and $y = 4$. Cross-sections taken perpendicular to the y-axis are squares.

(a) (1 Point) Sketch the region in the xy-plane that forms the base of the solid. Clearly label the intersection points of the curves $y = x^2$ and $y = 4$.

(b) (2 Points) Draw a typical slice of the cross-section of the solid and express its side length as a function of y.

(c) (3 Points) Set up, but do not evaluate, the definite integral that represents the volume of the solid.

Useful formulas

1. $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \ (n \neq -1)$
2. $\int \frac{1}{x} \, dx = \ln|x| + C$
3. $\int a^x \, dx = \frac{1}{\ln a} a^x + C$
4. $\int \ln x \, dx = x \ln x - x + C$
5. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \frac{1}{a} \arcsin\left(\frac{x}{a}\right) + C$
6. $\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$
7. $\int \cos ax \, dx = \frac{1}{a} \sin ax + C$
8. $\int \tan ax \, dx = -\frac{1}{a} \ln|\cos ax| + C$
9. $\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$
10. $\int \frac{1}{x\sqrt{x^2 + a^2}} \, dx = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + C$

Integration by Parts:

$$\int u dv = uv - \int v du \qquad \text{or} \qquad \int uv' dx = uv - \int vu' dx$$