AMAT112 Calculus I — Exam 3A — Spring 2025

Print Name:

UAlbany Email:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	10	
2	11	
3	18	
4	12	
5	10	
6	8	
Total	69	

YOUR SIGNATURE:

(Similar to Practice Assessment 11)

(1) (a) (5 Points) Find the linear approximation of $h(x) = 4\sin(2x)$ at $x = \dfrac{\pi}{2}$.

(b) (5 Points) Let $y = (x + x^2)^3$. Find the differential dy when $x = 1$ and $dx = 0.1$.

Your signature:

(Similar to Practice Assessment 12 and 13)

(2) (a) (5 Points) Consider the function $k(x) = -x^2 + 5x - 6$. Find all values of c in the interval $[1, 3]$ that satisfy the conclusion of the Mean Value Theorem for $k(x)$ on this interval.

(b) (6 Points) Find the absolute maximum and minimum values for the function $k(x) = -x^2 + 5x - 6$ in the closed interval $[1, 3]$.

YOUR SIGNATURE: ______________________________

(Similar to Practice Assessment 14)

(3) Consider the function $f(x)$ and its derivatives:

$$f(x) = x(x-5)^3, \quad f'(x) = (x-5)^2(4x-5), \quad f''(x) = 6(x-5)(2x-5)$$

Fill in the blanks below.

(a) (3 Points) The critical number(s) of f is (are)______________.

(b) (6 Points) f is decreasing on the interval(s) ____________________________ and

f is increasing on the interval(s) ____________________________.

(Problem 3 Continued) Consider the function $f(x)$ and its derivatives:

$f(x) = x(x-5)^3, \quad f'(x) = (x-5)^2(4x-5), \quad f''(x) = 6(x-5)(2x-5)$

(c) (3 Points) The point(s) of inflection of f is (are)_______________.

(d) (6 Points) f is concave up on the interval(s) _________________________ and

f is concave down on the interval(s) _________________________.

YOUR SIGNATURE:

(Similar to Practice Assessment 15)

(4) (4 Points Each) For each of the following limits determine which indeterminate form the expression corresponds to, then calculate the limit.

(a) $\displaystyle \lim_{x\to\infty} \frac{\sqrt{x}-5}{\sqrt{4x}+5}$

(b) $\displaystyle \lim_{x\to 0^+} \frac{\sin(x)}{x+x^{3/2}}$

(c) $\displaystyle \lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)$

(Similar to Practice Assessment 16)

(5) (10 Points) An airline's policy requires that all baggage must be box-shaped with a sum of length, width, and height equal to 69 inches. Determine the dimensions of a **square based** box with the maximum possible volume under this policy. *You must also use a derivative test to justify that your answer is a maximum.*

(Similar to Practice Assessment 17)

(6) (4 Points Each) Evaluate the indefinite integrals.

(a) $\displaystyle\int \frac{5 + x^2 + \sqrt{x}}{\sqrt{x}}\, dx$

(b) $\displaystyle\int \left(3^x - 2x + \sec^2(x)\right)\, dx$

Formulas you might find useful

- **The derivative of a function**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Some rules of differentiation**

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- **The equation of the tangent line to a function f for $x = a$ is given by**

$$y = f(a) + f'(a)(x - a)$$

- **The derivative of the inverse function f^{-1} at $x = a$ is given by**

$$\frac{d}{dx}\left(f^{-1}(x)\right)\Big|_{x=a} = \frac{1}{f'(f^{-1}(a))}.$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$