UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT112 Calculus I — Final Exam A — Spring 2025

Print Name:

UAlbany Email:

Please indicate your lecture section with a check mark (✓) in the leftmost column.

✓	Class No	Professor	Day and Time
	3432	Riley Decker	MWF 3:00-4:15PM
	5821	Mira Iskander	MWF 3:00-4:15PM
	5822	Sam Spellman	MWF 3:00-4:15PM
	5823	Lynn Greene	TTh 8:30-10:20AM
	5824	Susan Beckhardt	MWF 10:10-11:25AM
	5825	Peter Young	MWF 1:10-2:25PM
	5826	Michael Muller	MWF 1:10-2:25PM
	5827	Amber Ramey	MWF 10:10-11:25AM
	5828	Susan Beckhardt	MWF 8:00-9:15AM
	5829	Michael Muller	MWF 8:00-9:15AM

Directions: You have **120 minutes**. Show all necessary work as neatly and clearly as possible. No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points	Problem	Possible	Points
1	10		6	10	
2	9		7	14	
3	9		8	8	
4	6		9	8	
5	6		10**	8	
Total (Out of 80) =					

**Optional Extra Credit Problem

(1) (10 points) A car's fuel efficiency, E, in miles per gallon (mpg), is a function of its speed, s, in miles per hour (mph). This relationship is represented by the function $E = f(s)$.

(a) (2 points) Which of the following best explains the meaning of the statement $f(55) = 32$ in the context of the car's fuel efficiency? Circle your answer.

(i) A car traveling at 32 mph achieves a fuel efficiency of 55 mpg.

(ii) A car traveling at 55 mph achieves a fuel efficiency of 32 mpg.

(iii) The fuel efficiency of the car increases by 32 mpg for every 55 mph increase in speed.

(iv) The fuel efficiency of the car decreases by 32 mpg for every 55 mph increase in speed.

(v) The maximum fuel efficiency of the car is 32 mpg when traveling at 55 mph.

(b) (2 points) Which of the following best explains the meaning of the statement $f'(55) = -0.2$ in the context of the car's fuel efficiency? Circle your answer.

(i) When the car is traveling at 55 mph, its fuel efficiency is decreasing at a rate of 0.2 mpg per mph.

(ii) When the car's fuel efficiency is 55 mpg, its speed is decreasing at a rate of 0.2 mph per mpg.

(iii) The fuel efficiency of the car decreases by 0.2 mpg for every 55 mph increase in speed.

(iv) The fuel efficiency of the car is 0.2 mpg when the car is traveling at 55 mph.

(v) The average fuel efficiency of the car is -0.2 mpg.

(c) (6 points) Using the information from parts (a) and (b), use a linear approximation estimate the fuel efficiency of the car at 58 mph. Include the correct units in your answer. Show your work.

(2) Compute the derivatives of each of the following functions. You **do not** have to simplify your final answer.

(a) (3 Points) $f(x) = 5x^3 - 2\sqrt{x} + 7e$

(b) (3 Points) $g(x) = e^{4x}\cos(x)$

(c) (3 Points) $k(x) = \dfrac{x^2+1}{\ln(x)}$

(3) Consider the curve

$$y^2 + x^2y = 3x^2.$$

(a) (5 Points) Find an expression for $\dfrac{dy}{dx}$.

(b) (4 Points) Find the equation of the tangent line to the curve $y^2 + x^2y = 3x^2$ at the point $(2, 2)$.

(4) (2 Points Each) Compute the value of each limit.

(a) $\lim_{x \to -4} \frac{x^2 - 16}{x + 4}$

(b) $\lim_{x \to \frac{1}{3}^-} \frac{|3x - 1|}{3x - 1}$

(c) $\lim_{x \to 0^+} \ln(x)$

(5) (2 Points Each) Compute the value of each limit.

(a) $\displaystyle\lim_{x\to\infty} \frac{x^3 - 7x^2 + 6x - 5}{(2x-3)(x^2-5)}$

(b) $\displaystyle\lim_{x\to\infty} \frac{e^{2x}}{1+x^2}$

(c) $\displaystyle\lim_{x\to 0} \frac{\sin(3x)}{x}$

(6) (a) (6 Points) Let $P(x)$ be a piecewise function defined by

$$P(x) = \begin{cases} x^k & \text{if } x \leq 2 \\ \\ 10 - x & \text{if } x > 2. \end{cases}$$

The function P is continuous everywhere, if $k =$ ____________.

(b) (4 Points) If $q(x) = x^2 - 5x + 4$, what input c satisfies the conclusion of the Mean Value Theorem applied to $q(x)$ on the interval $[0, 3]$?

(7) Consider the function $f(x) = x^3 - 6x^2 + 9x + 1$. Fill in the blanks below.

(a) (3 Points) The critical number(s) of f is (are)____________.

(b) (4 Points) f is decreasing on the interval(s) ____________________ and

f is increasing on the interval(s) ____________________.

(Problem 7 Continued)
Consider the function $f(x) = x^3 - 6x^2 + 9x + 1$. *Fill in the blanks below.*

(c) (3 Points) The point(s) of inflection of f is (are)______________.

(d) (4 Points) f is concave up on the interval(s) ___________________________ and

f is concave down on the interval(s) ___________________________.

(8) (8 Points) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 12 - x^2$. What are the dimensions of such a rectangle with the greatest possible area? *Clearly justify that your answer is a maximum.*

(9) (4 Points Each) Evaluate the indefinite integrals.

(a) $\displaystyle\int \left(4x^3 - 6x + 5\right)\, dx$

(b) $\displaystyle\int (3e^x + \sin(x))\, dx$

Optional Extra Credit Problems

(10) (a) (4 Points) Use geometry to evaluate the integral below.

$$\int_0^4 (|2-x|+3)\,dx$$

(b) A table of values of a function f is given below.

x	0	9	18	27	36
$f(x)$	10	-15	25	-5	-10

(4 Points) Find R_4, a right-endpoint approximation with $n = 4$ for $\displaystyle\int_0^{36} f(x)dx$.

Formulas you might find useful

- **The derivative of a function**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Some rules of differentiation**

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- **The equation of the tangent line to a function f for $x = a$ is given by**

$$y = f(a) + f'(a)(x-a)$$

- **The derivative of the inverse function f^{-1} at $x = a$ is given by**

$$\frac{d}{dx}\left(f^{-1}(x)\right)\bigg|_{x=a} = \frac{1}{f'(f^{-1}(a))}.$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$