

# AMAT 108 ELEMENTARY STATISTICS SPRING 2025

# FINAL EXAM VERSION 1

Print Name: \_\_\_\_\_ UAlbany Email: \_\_\_\_\_

**Directions:** You have **120 minutes** to answer the following questions. *No notes, textbooks, mobile phones or other aids are allowed. Only scientific calculators are allowed.* For all multiple-choice questions, select **one** answer from among the choices given. No explanation is required to be shown and no partial credit will be given. Make sure to **completely** fill in the circle corresponding to your chosen answer. For all free-response questions, you **must** show all necessary work to receive full credit. An answer with no work, even if correct, will not receive full credit. Please circle or box your final answer. All work, if needed, is to be rounded to **five** decimal places.

**Do not detach any pages.** Please choose your section with a check mark (✓) in the left-most column.

✓	Section	Instructor Name	Meeting Time	Meeting Days	Meeting Location
	1651	John Habib	11:40AM	M/W	HU 124
	3998	Seth Hulbert	9:00AM	T/TH	SS 116
	4046		12:00PM		HU 124
	6998	Tung Lam	9:00AM	T/TH	FA 126
	2761		1:30PM		TA 118
	1648	James Lamatina	1:10PM	M/W	LC 2
	3402		3:00PM		LC 25
	3209		9:00AM	T/TH	LC 3B
	1649	Chris Lange	3:00PM	T/TH	SS 255
	1653		4:30PM		
	1652	Douglas Rosenberg	3:00PM	M/W	BB B006
	1654		4:30PM		HU 133
	1650	Sam Spellman	1:10PM	M/W	FA 126
	3382	Alea Wittig	10:30AM	T/TH	HU 123
	3399		12:00PM		HU 129
	3508	Peter Young	8:00AM	M/W	SS 255
	3406		11:40AM		HU 123

**Exam Scoring:**

Questions	Possible Points	Points Earned
1-10	10	
11	8	
12	5	
13	8	
14	8	
15	7	
16	14	
17	12	
Total Points	72	
Percentage		

1. Suppose  $X$  is a discrete random variable that has possible values 6, 11, and 15. Let  $P(X = 6) = 0.81$  and  $P(X = 11) = 0.07$ . Which of the following is equal to  $P(X = 15)$ ? (1 pt.)

- ① 0.88
- ② 0.81
- ③ 0.12
- ④ 0.07
- ⑤ None of the previous options.

**Questions 2-5 are based on the following: Suppose  $A$ ,  $B$ ,  $D$ , and  $E$  are events with  $P(A) = 0.41$ ,  $P(B) = 0.36$ ,  $P(D) = 0.12$ , and  $P(E) = 0.09$ .**

2. Suppose  $B$  and  $D$  are disjoint events. Which of the following is equal to  $P(B \cup D)$ ? (1 pt.)

- ① 0.0432
- ② 0.53
- ③ 0.21
- ④ 0.48
- ⑤ None of the previous options.

3. Now suppose  $B$  and  $D$  are independent events, not disjoint events. Which of the following is equal to  $P(B \cap D)$ ? (1 pt.)

- ① 0.0432
- ② 0.53
- ③ 0.21
- ④ 0.48
- ⑤ None of the previous options.

4. Given that  $P(A \cap E) = 0.0583$ , which of the following is equal to  $P(A|E)$ ? (1 pt.)

- ① 0.14220
- ② 0.64778
- ③ 0.09
- ④ 1.54374
- ⑤ 0.41

5. Which of the following is equal to  $P(A^C)$ ? (1 pt.)

- ① 0.59
- ② 0.71930
- ③ 0.57
- ④ 0.65779
- ⑤ None of the previous options.

**Questions 6 and 7 are based on the following: Suppose  $Z$  has a standard normal distribution.**

6. Correct to four decimal places, which of the following is equal to  $P(-2.74 \leq Z \leq 1.45)$ ? (1 pt.)

① 0.9234

④ 0.9296

② 0.9265

⑤ None of the previous options.

③ 0.0031

7. The closest value of  $c$  so that  $P(Z \leq c) \approx 0.0179$  is... (1 pt.)

① -2.11

④ -2.93

② -2.94

⑤ -2.10

③ -2.92

**Questions 8 and 9 are based on the following: A bin contains five red balls, six green balls, and ten black balls.**

8. Professor Rosenberg selects two balls from the bin with replacement. The probability that Professor Rosenberg selects a green ball second, given that he selects a black ball first, equals... (1 pt.)

① 0.5

④ 0.47619

② 0.28571

⑤ None of the previous options.

③ 0.3

9. Professor Habib selects two balls from the bin without replacement. The probability that Professor Habib selects a green ball second, given that he selects a black ball first, equals... (1 pt.)

① 0.5

④ 0.47619

② 0.28571

⑤ None of the previous options.

③ 0.3

10. Suppose  $n = 51$  and  $p$  are such that all requirements for both confidence intervals are met. Which of the following is an appropriate 95%  $t^*$  critical number? (1 pt.)

① 2.109

④ 2.000

② 1.960

⑤ 1.676

③ 2.009

11. The discrete random variable  $X$  has probability distribution given in the table below:

$X$	13	21	23
$p(X)$	0.61	0.32	0.07

(a) Find the mean value of  $X$ . Do not round your answer. (3 pts.)

(b) Find the standard deviation of  $X$ . Round your answer to *three* decimal places. (5 pts.)

12. Suppose  $Y$  has a uniform distribution on the interval  $(4, 27)$ .

(a) Find the height of the density curve. *Leave your answer in fractional form.* (2 pts.)

(b) Find the probability that  $Y$  is greater than or equal to 16. Round your answer to *five* decimal places. (3 pts.)

13. Suppose it is a widely-held belief that the distribution of the number of cats born per litter is normal with mean 5.026 and standard deviation 1.438. Professor Wittig selects 206 cat litters at random and records the number of cats in the litter.
- (a) Compute the probability that the average number of cats born in the 206 litters Professor Wittig selected is less than or equal to 4.713. *Hint:* The distribution of the average number of cats born in the 206 litters Professor Wittig selected is normal. (4 pts.)
- (b) Compute the probability that the average number of cats born in the 206 litters Professor Wittig selected is between 4.713 and 5.339 inclusive. *Hint:* This is an example of a symmetric-limits problem. (4 pts.)

14. Professor Spellman is studying the lifespan of nuclear power plants. An analysis of 74 nuclear power plants shows that 23% of them are decommissioned within the first twenty-five years.
- (a) Professor Spellman wishes to construct a central 96% confidence interval for the proportion of all nuclear power plants that are decommissioned within the first twenty-five years. Find an appropriate critical number. (1 pt.)
- (b) Construct a central 96% confidence interval for the proportion of all nuclear power plants that are decommissioned within the first twenty-five years. Round *both* endpoints to *three* decimal places. (5 pts.)
- (c) Find a conservative estimate for the sample size required to estimate the proportion of all nuclear power plants that are decommissioned within the first twenty-five years to within 0.008 with 96% confidence. (3 pts.)

15. The Math Department at SUNY Albany is investigating the number of staples used for student exams. The University believes that the Department uses an average of 17936 staples per semester. However, the Department Chair thinks the University's belief is an underestimate. To assess this, the Chair asks Professors Hulbert, Lam, and Lamatina to randomly select 101 semesters and count the number of staples used. Their analysis found an average of 18548 staples used per semester, with a standard deviation of 3058 staples. The Chair wishes to know if there is enough evidence to conclude that the University's belief is an underestimate.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

①  $H_0: \mu = 17936$   
vs.  
 $H_1: \mu < 17936$

②  $H_0: \mu = 17936$   
vs.  
 $H_1: \mu > 17936$

③  $H_0: \mu = 17936$   
vs.  
 $H_1: \mu \neq 17936$

(b) Compute the  $t^*$  test statistic. Round your answer to *one* decimal place. (3 pts.)

(c) The test statistic has a  $t$  distribution with how many degrees of freedom? (1 pt.)

① 102

② 100

③ 99

④ 101

(d) Compute the  $p$ -value of the test, and express your answer to *three* decimal places. (2 pts.)

(e) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject  $H_0$  at  $\alpha = 0.05$ . There is not enough evidence to conclude that the University's belief is an underestimate.

② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . There is a slight amount of evidence to conclude that the University's belief is an underestimate.

③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . There is a convincing amount of evidence to conclude that the University's belief is an underestimate.

④ Reject  $H_0$  at  $\alpha = 0.001$ . There is an overwhelming amount of evidence to conclude that the University's belief is an underestimate.



16. Recently, the State of New York has been exploring ways to reduce its energy expenses. One such initiative, championed by Governor Hochul, involves converting all incandescent light bulbs to LED light bulbs. The manufacturer of a certain brand of LED light bulb asserts that their bulbs are more likely to last at least five years than incandescent ones. Before entering into a supply contract with the manufacturer on behalf of the State, however, Governor Hochul asks Professor Young to conduct an experiment comparing both types of light bulbs. The objective is to determine if the LED light bulbs are more likely to last at least five years than the incandescent ones. Summary statistics from Professor Young's experiment are shown below:

Type of Light Bulb	Incandescent	LED Bulb
Number of Bulbs	12537	12413
Number of Successful Bulbs	6354	6498
Proportion of Successful Bulbs	50.682%	52.348%

In his final report to Governor Hochul, Professor Young wishes to use the 2-Sample  $Z$  Test for Equal Proportions to assess whether there is sufficient evidence to conclude that the proportion of all LED light bulbs that are successful is higher than the proportion of all incandescent light bulbs that are successful. Let  $p_1$  represent the proportion of all incandescent light bulbs that are successful, and let  $p_2$  denote the proportion of all LED light bulbs that are successful.

- (a) Select the correct pair of statistical hypotheses. (1 pt.)

①  $H_0: p_1 = p_2$   
vs.  
 $H_1: p_1 < p_2$

②  $H_0: p_1 = p_2$   
vs.  
 $H_1: p_1 > p_2$

③  $H_0: p_1 = p_2$   
vs.  
 $H_1: p_1 \neq p_2$

- (b) Are all requirements for the 2-Sample  $Z$  Test for Equal Proportions met? (1 pt.)

- ① Yes, because both populations have normal distributions.  
② Yes, because both sample sizes are sufficiently large.  
③ Yes, because both sets of success-failure conditions are met.  
④ No, because none of the above are true.

- (c) Compute the pooled sample proportion. Round your answer to *five* decimal places. (3 pts.)

- (d) Compute the  $z^*$  test statistic. Round your answer to *two* decimal places, where appropriate. (5 pts.)

- (e) Compute the  $p$ -value of the test, and express your answer to *four* decimal places. (3 pts.)

- (f) Which of the following is the correct conclusion? (1 pt.)

- ① Fail to reject  $H_0$  at  $\alpha = 0.05$ . Professor Young does not have enough evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.
- ② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . Professor Young has a slight amount of evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.
- ③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . Professor Young has a convincing amount of evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.
- ④ Reject  $H_0$  at  $\alpha = 0.001$ . Professor Young has an overwhelming amount of evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.

17. In a weekly coordination meeting towards the beginning of the Spring 2025 semester, Professor Medina shared his belief with Professor Lange that student success in AMAT 108 is linked to the number of Brightspace announcements made by instructors. Professor Lange, the Spring 2025 AMAT 108 exam writer, suggested testing this claim during the current semester. They agreed on a study involving two groups of AMAT 108 sections: one with at least eight Brightspace announcements and another with fewer than eight. The data collected were Exam 2 scores out of 100, which leads to the summary statistics below:

# Announcements	< 8	$\geq 8$
Average	71.615	82.928
Standard Deviation	25.305	15.637
Number of Students	118	120

For the purpose of this problem, assume the scores in both samples are drawn from normally-distributed populations. Professor Lange and Professor Medina wish to use Welch's  $T$  Test to see if there is enough evidence to support Professor Medina's claim that significant differences exist in student performance based on the number of Brightspace announcements made. Let  $\mu_1$  be the average score in sections with fewer than eight announcements and  $\mu_2$  be the average score in sections with at least eight announcements.

- (a) Select the correct pair of statistical hypotheses. (1 pt.)

① $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 < \mu_2$	② $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 > \mu_2$	③ $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$
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- (b) Are all requirements for Welch's  $T$  Test met? (1 pt.)

- ① Yes, because both populations have normal distributions.
- ② Yes, because both samples have normal distributions.
- ③ Yes, because both sample sizes are sufficiently large.
- ④ Yes, because the requirements for the One-Sample  $T$  Test for  $\mu$  are met for each sample.
- ⑤ No, because none of the above are true.

- (c) Compute the  $t^*$  test statistic. Round your answer to *one* decimal place. (5 pts.)

(d) The test statistic has a  $t$  distribution with how many degrees of freedom? (1 pt.)

① 113

⑤ 117

② 114

⑥ 118

③ 115

⑦ 119

④ 116

⑧ 120

(e) Compute the  $p$ -value of the test, and express your answer to *three* decimal places. (3 pts.)

(f) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject  $H_0$  at  $\alpha = 0.05$ . There is not enough evidence to conclude that Professor Medina's claim is correct.

② Reject  $H_0$  at  $\alpha = 0.05$ , but fail to reject  $H_0$  at  $\alpha = 0.01$ . There is a slight amount of evidence to conclude that Professor Medina's claim is correct.

③ Reject  $H_0$  at  $\alpha = 0.01$ , but fail to reject  $H_0$  at  $\alpha = 0.001$ . There is a convincing amount of evidence to conclude that Professor Medina's claim is correct.

④ Reject  $H_0$  at  $\alpha = 0.001$ . There is an overwhelming amount of evidence to conclude that Professor Medina's claim is correct.

**Formula Sheet:**

- Probability:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) \text{ if events } A \text{ and } B \text{ are disjoint}$$

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B) \text{ if events } A \text{ and } B \text{ are independent}$$

- Complement probability (probability complement rule):  $P(A^C) = 1 - P(A)$
- Conditional probability of event  $A$  given event  $B$  occurs:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- At least 1 Rule:  $P(\text{at least 1 success in } n \text{ trials}) = 1 - P(\text{no successes in } n \text{ trials})$
- Finding the height of a uniform distribution:

$$\text{Height} = \frac{1}{\text{Base}} = \frac{1}{b - a}$$

- Probability (area) of a uniform distribution: Probability (or area) = Base  $\cdot$  Height
- Mean of discrete random variable  $X$  with possible values  $x_1, x_2, \dots, x_n$ :

$$\mu_X = E(X) = x_1p(x_1) + x_2p(x_2) + \dots + x_np(x_n)$$

- Standard deviation of a discrete random variable  $X$  with possible values  $x_1, x_2, \dots, x_n$ :

$$\sigma_X = \sqrt{(x_1 - \mu_X)^2p(x_1) + (x_2 - \mu_X)^2p(x_2) + \dots + (x_n - \mu_X)^2p(x_n)}$$

or

$$\sigma_X = \sqrt{E(X^2) - (E(X))^2}$$

- Mean and standard deviation for sampling distribution of  $\bar{X}$ :

$$\mu_{\bar{X}} = \mu \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

where  $\mu$  = population mean and  $\sigma$  = population standard deviation

- Standardized variable ( $z$ -score) for  $\bar{X}$  (when  $\sigma$  known):

$$Z = \frac{\bar{X} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

- Mean and standard deviation for sampling distribution of  $\hat{p}$ :

$$\mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

where  $p$  = population proportion of successes

- Sample proportion of successes:

$$\hat{p} = \frac{\text{number of successes in the sample}}{n}$$

- Standardized variable ( $z$ -score) for  $\hat{p}$ :

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- Success/failure condition for the sampling distribution of  $\hat{p}$  to be approximately normal:

$$np \geq 10 \quad \text{and} \quad n(1-p) \geq 10$$

- Confidence interval for a population proportion:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Determining the sample size needed to be within  $M$  = margin of error under a certain confidence level:

$$n = p(1-p) \left( \frac{z^*}{M} \right)^2$$

- To find the conservatively large sample size needed, set  $p = 0.5$ .

- Confidence interval for a population mean (when  $\sigma$  unknown):

$$\bar{x} \pm t^* \left( \frac{s}{\sqrt{n}} \right) \quad df = n - 1$$

- $z^*$  test statistic for hypothesis testing of a population proportion:

$$z^* = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- $t^*$  test statistic for hypothesis testing of a population mean (when  $\sigma$  unknown):

$$t^* = \frac{\bar{x} - \mu}{\left( \frac{s}{\sqrt{n}} \right)} \quad df = n - 1$$

- Hypothesis testing for the difference of two means (Welch's  $T$  Test):

$$t^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad df = \text{smaller of } n_1 - 1 \text{ and } n_2 - 1$$

- Hypothesis testing for the difference of two proportions (2-Sample  $Z$  Test for Equal Proportions):

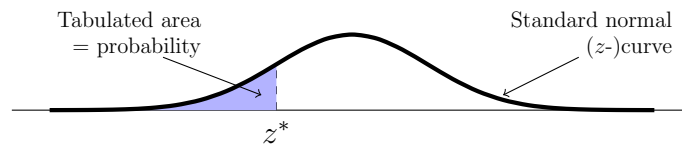
$$z^* = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}}$$

- Combined/pooled proportion:

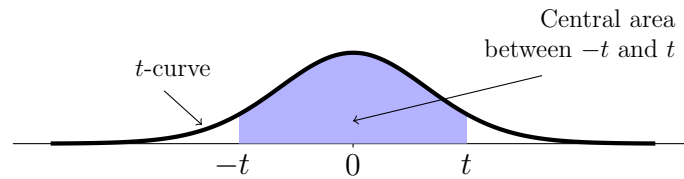
$$\hat{p}_c = \frac{n_1 \cdot \hat{p}_1 + n_2 \cdot \hat{p}_2}{n_1 + n_2} \quad \text{or} \quad \hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

- Success/failure condition for the 2-Sample  $Z$  Test for Equal Proportions:

$$n_1 \hat{p}_1 \geq 10 \quad n_1(1 - \hat{p}_1) \geq 10 \quad n_2 \hat{p}_2 \geq 10 \quad n_2(1 - \hat{p}_2) \geq 10$$

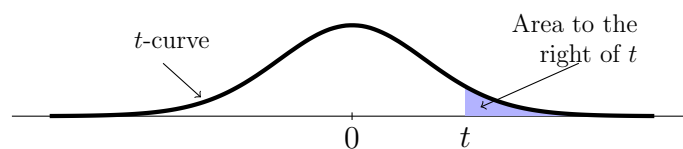
***Standard Normal Table***

$z^*$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

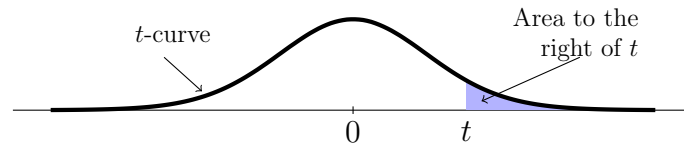
**t-Distribution Table of Critical Values**

degree of freedom	Central Area Captured / Confidence Level								
	80%	90%	95%	96%	97%	98%	99%	99.8%	99.9%
<b>1</b>	3.078	6.314	12.706	15.895	21.205	31.821	63.657	318.309	636.619
<b>2</b>	1.886	2.920	4.303	4.849	5.643	6.965	9.925	22.327	31.599
<b>3</b>	1.638	2.353	3.182	3.482	3.896	4.541	5.841	10.215	12.924
<b>4</b>	1.533	2.132	2.776	2.999	3.298	3.747	4.604	7.173	8.610
<b>5</b>	1.476	2.015	2.571	2.757	3.003	3.365	4.032	5.893	6.869
<b>6</b>	1.440	1.943	2.447	2.612	2.829	3.143	3.707	5.208	5.959
<b>7</b>	1.415	1.895	2.365	2.517	2.715	2.998	3.499	4.785	5.408
<b>8</b>	1.397	1.860	2.306	2.449	2.634	2.896	3.355	4.501	5.041
<b>9</b>	1.383	1.833	2.262	2.398	2.574	2.821	3.250	4.297	4.781
<b>10</b>	1.372	1.812	2.228	2.359	2.527	2.764	3.169	4.144	4.587
<b>11</b>	1.363	1.796	2.201	2.328	2.491	2.718	3.106	4.025	4.437
<b>12</b>	1.356	1.782	2.179	2.303	2.461	2.681	3.055	3.930	4.318
<b>13</b>	1.350	1.771	2.160	2.282	2.436	2.650	3.012	3.852	4.221
<b>14</b>	1.345	1.761	2.145	2.264	2.415	2.624	2.977	3.787	4.140
<b>15</b>	1.341	1.753	2.131	2.249	2.397	2.602	2.947	3.733	4.073
<b>16</b>	1.337	1.746	2.120	2.235	2.382	2.583	2.921	3.686	4.015
<b>17</b>	1.333	1.740	2.110	2.224	2.368	2.567	2.898	3.646	3.965
<b>18</b>	1.330	1.734	2.101	2.214	2.356	2.552	2.878	3.610	3.922
<b>19</b>	1.328	1.729	2.093	2.205	2.346	2.539	2.861	3.579	3.883
<b>20</b>	1.325	1.725	2.086	2.197	2.336	2.528	2.845	3.552	3.850
<b>21</b>	1.323	1.721	2.080	2.189	2.328	2.518	2.831	3.527	3.819
<b>22</b>	1.321	1.717	2.074	2.183	2.320	2.508	2.819	3.505	3.792
<b>23</b>	1.319	1.714	2.069	2.177	2.313	2.500	2.807	3.485	3.768
<b>24</b>	1.318	1.711	2.064	2.172	2.307	2.492	2.797	3.467	3.745
<b>25</b>	1.316	1.708	2.060	2.167	2.301	2.485	2.787	3.450	3.725
<b>26</b>	1.315	1.706	2.056	2.162	2.296	2.479	2.779	3.435	3.707
<b>27</b>	1.314	1.703	2.052	2.158	2.291	2.473	2.771	3.421	3.690
<b>28</b>	1.313	1.701	2.048	2.154	2.286	2.467	2.763	3.408	3.674
<b>29</b>	1.311	1.699	2.045	2.150	2.282	2.462	2.756	3.396	3.659
<b>30</b>	1.310	1.697	2.042	2.147	2.278	2.457	2.750	3.385	3.646
<b>40</b>	1.303	1.684	2.021	2.123	2.250	2.423	2.704	3.307	3.551
<b>50</b>	1.299	1.676	2.009	2.109	2.234	2.403	2.678	3.261	3.496
<b>60</b>	1.296	1.671	2.000	2.099	2.223	2.390	2.660	3.232	3.460
<b>70</b>	1.294	1.667	1.994	2.093	2.215	2.381	2.648	3.211	3.435
<b>80</b>	1.292	1.664	1.990	2.088	2.209	2.374	2.639	3.195	3.416
<b>90</b>	1.291	1.662	1.987	2.084	2.205	2.368	2.632	3.183	3.402
<b>100</b>	1.290	1.660	1.984	2.081	2.201	2.364	2.626	3.174	3.390
<b>110</b>	1.289	1.659	1.982	2.078	2.199	2.361	2.621	3.166	3.381
<b>120</b>	1.289	1.658	1.980	2.076	2.196	2.358	2.617	3.160	3.373
<b><math>z</math>-critical = <math>\infty</math></b>	1.282	1.645	1.960	2.054	2.170	2.326	2.576	3.090	3.291



**Tail Areas for  $t$  Curves**

$t^* \backslash df$	1	2	...	99	100	101	102	...	109	110
0.0	0.500	0.500	...	0.500	0.500	0.500	0.500	...	0.500	0.500
0.1	0.468	0.465	...	0.460	0.460	0.460	0.460	...	0.460	0.460
0.2	0.437	0.430	...	0.421	0.421	0.421	0.421	...	0.421	0.421
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.9	0.267	0.232	...	0.185	0.185	0.185	0.185	...	0.185	0.185
1.0	0.250	0.211	...	0.160	0.160	0.160	0.160	...	0.160	0.160
1.1	0.235	0.193	...	0.137	0.137	0.137	0.137	...	0.137	0.137
1.2	0.221	0.177	...	0.117	0.116	0.116	0.116	...	0.116	0.116
1.3	0.209	0.162	...	0.098	0.098	0.098	0.098	...	0.098	0.098
1.4	0.197	0.148	...	0.082	0.082	0.082	0.082	...	0.082	0.082
1.5	0.187	0.136	...	0.068	0.068	0.068	0.068	...	0.068	0.068
1.6	0.178	0.125	...	0.056	0.056	0.056	0.056	...	0.056	0.056
1.7	0.169	0.116	...	0.046	0.046	0.046	0.046	...	0.046	0.046
1.8	0.161	0.107	...	0.037	0.037	0.037	0.037	...	0.037	0.037
1.9	0.154	0.099	...	0.030	0.030	0.030	0.030	...	0.030	0.030
2.0	0.148	0.092	...	0.024	0.024	0.024	0.024	...	0.024	0.024
2.1	0.141	0.085	...	0.019	0.019	0.019	0.019	...	0.019	0.019
2.2	0.136	0.079	...	0.015	0.015	0.015	0.015	...	0.015	0.015
2.3	0.131	0.074	...	0.012	0.012	0.012	0.012	...	0.012	0.012
2.4	0.126	0.069	...	0.009	0.009	0.009	0.009	...	0.009	0.009
2.5	0.121	0.065	...	0.007	0.007	0.007	0.007	...	0.007	0.007
2.6	0.117	0.061	...	0.005	0.005	0.005	0.005	...	0.005	0.005
2.7	0.113	0.057	...	0.004	0.004	0.004	0.004	...	0.004	0.004
2.8	0.109	0.054	...	0.003	0.003	0.003	0.003	...	0.003	0.003
2.9	0.106	0.051	...	0.002	0.002	0.002	0.002	...	0.002	0.002
3.0	0.102	0.048	...	0.002	0.002	0.002	0.002	...	0.002	0.002
3.1	0.099	0.045	...	0.001	0.001	0.001	0.001	...	0.001	0.001
3.2	0.096	0.043	...	0.001	0.001	0.001	0.001	...	0.001	0.001
3.3	0.094	0.040	...	0.001	0.001	0.001	0.001	...	0.001	0.001
3.4	0.091	0.038	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.5	0.089	0.036	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.6	0.086	0.035	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.7	0.084	0.033	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.8	0.082	0.031	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.9	0.080	0.030	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.0	0.078	0.029	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.1	0.076	0.027	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.2	0.074	0.026	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.3	0.073	0.025	...	0.000	0.000	0.000	0.000	...	0.000	0.000

**Tail Areas for  $t$  Curves**

$t^* \backslash df$	111	112	113	114	115	116	117	118	119	120
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460
0.2	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.9	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
1.0	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
1.1	0.137	0.137	0.137	0.137	0.137	0.137	0.137	0.137	0.137	0.137
1.2	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116
1.3	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098
1.4	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082
1.5	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068
1.6	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056
1.7	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
1.8	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
1.9	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
2.0	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
2.1	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019
2.2	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
2.3	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012
2.4	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
2.5	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
2.6	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
2.7	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.8	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000