UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT 108 Elementary Statistics — Final Exam
Spring 2025 — Version 1

Answer Key

Directions: You have **120 minutes** to answer the following questions. ***No notes, textbooks, mobile phones or other aids are allowed. Only scientific calculators are allowed.*** For all multiple-choice questions, select **one** answer from among the choices given. No explanation is required to be shown and no partial credit will be given. Make sure to **completely** fill in the circle corresponding to your chosen answer. For all free-response questions, you **must** show all necessary work to receive full credit. An answer with no work, even if correct, will not receive full credit. Please circle or box your final answer. All work, if needed, is to be rounded to ***five*** decimal places.

Do not detach any pages. Please choose your section with a check mark (✓) in the left-most column.

✓	Instructor Name	Meeting Time	Meeting Days	Meeting Location	Section
	John Habib	11:40AM	M/W	HU 124	1651
	Seth Hulbert	09:00AM	T/TH	SS 116	3998
		12:00PM		HU 124	4046
	Tung Lam	09:00AM	T/TH	FA 126	6998
		01:30PM		TA 118	2761
	James Lamatina	01:10PM	M/W	LC 2	1648
		03:00PM		LC 25	3402
		09:00AM	T/TH	LC 3B	3209
	Chris Lange	03:00PM	T/TH	SS 255	1649
		04:30PM			1653
	Douglas Rosenberg	03:00PM	M/W	BB B006	1652
		04:30PM		HU 133	1654
	Sam Spellman	01:10PM	M/W	FA 126	1650
	Alea Wittig	10:30AM	T/TH	HU 123	3382
		12:00PM		HU 129	3399
	Peter Young	08:00AM	M/W	SS 255	3508
		11:40AM		HU 123	3406

1. Suppose X is a discrete random variable that has possible values 6, 11, and 15. Let $P(X = 6) = 0.81$ and $P(X = 11) = 0.07$. Which of the following is equal to $P(X = 15)$? (1 pt.)

① 0.88
② 0.81
③ 0.12 (marked)
④ 0.07
⑤ None of the previous options.

Questions 2-5 are based on the following: Suppose A, B, D, and E are events with $P(A) = 0.41$, $P(B) = 0.36$, $P(D) = 0.12$, and $P(E) = 0.09$.

2. Suppose B and D are disjoint events. Which of the following is equal to $P(B \cup D)$? (1 pt.)

① 0.0432
② 0.53
③ 0.21
④ 0.48 (marked)
⑤ None of the previous options.

3. Now suppose B and D are independent events, not disjoint events. Which of the following is equal to $P(B \cap D)$? (1 pt.)

① 0.0432 (marked)
② 0.53
③ 0.21
④ 0.48
⑤ None of the previous options.

4. Given that $P(A \cap E) = 0.0583$, which of the following is equal to $P(A|E)$? (1 pt.)

① 0.14220
② 0.64778 (marked)
③ 0.09
④ 1.54374
⑤ 0.41

5. Which of the following is equal to $P(A^C)$? (1 pt.)

① 0.59 (marked)
② 0.71930
③ 0.57
④ 0.65779
⑤ None of the previous options.

Questions 6 and 7 are based on the following: Suppose Z has a standard normal distribution.

6. Correct to four decimal places, which of the following is equal to $P(-2.74 \leq Z \leq 1.45)$? (1 pt.)

① 0.9234
② 0.9265
③ 0.0031
④ 0.9296
⑤ None of the previous options.

7. The closest value of c so that $P(Z \leq c) \approx 0.0179$ is... (1 pt.)

① -2.11
② -2.94
③ -2.92
④ -2.93
⑤ -2.10

Questions 8 and 9 are based on the following: A bin contains five red balls, six green balls, and ten black balls.

8. Professor Rosenberg selects two balls from the bin with replacement. The probability that Professor Rosenberg selects a green ball second, given that he selects a black ball first, equals... (1 pt.)

① 0.5
② 0.28571
③ 0.3
④ 0.47619
⑤ None of the previous options.

9. Professor Habib selects two balls from the bin without replacement. The probability that Professor Habib selects a green ball second, given that he selects a black ball first, equals... (1 pt.)

① 0.5
② 0.28571
③ 0.3
④ 0.47619
⑤ None of the previous options.

10. Suppose $n = 51$ and p are such that all requirements for both confidence intervals are met. Which of the following is an appropriate 95% t^* critical number? (1 pt.)

① 2.109
② 1.960
③ 2.009
④ 2.000
⑤ 1.676

11. The discrete random variable X has probability distribution given in the table below:

X	13	21	23
$p(X)$	0.61	0.32	0.07

(a) Find the mean value of X. Do not round your answer. (3 pts.)

(1) $$\mu = 13(.61) + 21(.32) + 23(.07)$$
(2) $$= 7.93 + 6.72 + 1.61$$
(3) $$= 16.26$$

- $\boxed{+1}$ for products shown in (1) (no partial credit)
- $\boxed{+1}$ for adding all products together
- $\boxed{+1}$ for (3)
- $\boxed{-2}$ if the student shows

(4) $$\mu = \frac{13 + 21 + 23}{3}$$
(5) $$= 19$$

- $\boxed{-3}$ if the student shows

(6) $$\mu = \frac{.61 + .32 + .07}{3}$$
(7) $$\approx 0.333$$

(b) Find the standard deviation of X. Round your answer to *three* decimal places. (5 pts.) **If the student uses the standard method...**

(8) $$\sigma = \sqrt{.61(13 - 16.26)^2 + .32(21 - 16.26)^2 + .07(23 - 16.26)^2}$$
(9) $$\approx \sqrt{6.48284 + 7.18963 + 3.17993}$$
(10) $$= \sqrt{16.8524}$$
(11) $$\approx 4.105$$

- $\boxed{+1}$ for showing all squared deviations from the mean (no partial credit)
- $\boxed{+1}$ for multiplying each squared deviation by associated probability (no partial credit)
- $\boxed{+1}$ for adding all products together
- $\boxed{+1}$ for taking a square root
- $\boxed{+1}$ for (11) with correct rounding

If the student uses the simple calculation method...

(12) $$E(X^2) = 13^2(.61) + 21^2(.32) + 23^2(.07)$$
(13) $$= 281.24$$
(14) $$\sigma = \sqrt{281.24 - 16.26^2}$$
(15) $$\approx 4.105$$

- $\boxed{+1}$ each for (12), (13), and (15) with correct rounding (no partial credit)
- $\boxed{+2}$ for (14) (one point for difference inside square root and one point for taking square root)

Note. Follow any mistakes the student makes in (a) above.

12. Suppose Y has a uniform distribution on the interval $(4, 27)$.

(a) Find the height of the density curve. *Leave your answer in fractional form.* (2 pts.)

(16) $$f(y) = \frac{1}{27 - 4}$$

(17) $$= \frac{1}{23}$$

$$\downarrow\downarrow$$

(18) $$\text{height} = \frac{1}{23}$$

- $\boxed{+1}$ for difference of 27 and 4 as shown in (16)
- $\boxed{+1}$ for (18)

Note.

- If the student shows (17) but not (18), do not deduct any points.
- If the student expresses their answer as a decimal, deduct one point.

(b) Find the probability that Y is greater than or equal to 16. Round your answer to *five* decimal places. (3 pts.)

(19) $$P(Y \geq 16) = \frac{1}{23}(23 - 16)$$

(20) $$= \frac{23 - 16}{23}$$

(21) $$= \frac{7}{23}$$

(22) $$\approx 0.30435$$

- $\boxed{+1}$ for numerator in (20)
- $\boxed{+1}$ for denominator in (20)
- $\boxed{+1}$ for (22) with correct rounding

Note.

- If the student shows (19) but not (20), award one point for the contents of the parentheses and one point for the fraction on the outside.
- Students need not show (21) to receive full credit.
- If the student gives $16/23 \approx 0.69565$ as their answer, deduct one point.
- Follow any mistake the student made in (a) above.

13. Suppose it is a widely-held belief that the distribution of the number of cats born per litter is normal with mean 5.026 and standard deviation 1.438. Professor Wittig selects 206 cat litters at random and records the number of cats in the litter.

(a) Compute the probability that the average number of cats born in the 206 litters Professor Wittig selected is less than or equal to 4.713. *Hint:* The distribution of the average number of cats born in the 206 litters Professor Wittig selected is normal. (4 pts.)

$$P(\overline{X} \le 4.713) = P\left(\frac{\overline{X} - 5.026}{\left(\frac{1.438}{\sqrt{206}}\right)} \le \frac{4.713 - 5.026}{\left(\frac{1.438}{\sqrt{206}}\right)}\right) \tag{23}$$

$$\approx P(Z \le -3.12) \tag{24}$$

$$\approx 0.0009 \tag{25}$$

- +1 for showing the required conversion in (23)
- +1 for changing $\overline{X}$ to Z in (24)
- +1 for correct z-score calculation in (24) with correct rounding
- +1 for (25)

(b) Compute the probability that the average number of cats born in the 206 litters Professor Wittig selected is between 4.713 and 5.339 inclusive. *Hint:* This is an example of a symmetric-limits problem. (4 pts.) **If the student showed the usual method...**

$$P(4.713 \le \overline{X} \le 5.339) = P(-3.12 \le Z \le 3.12) \tag{26}$$

$$= P(Z \le 3.12) - P(Z \le -3.12) \tag{27}$$

$$\approx 0.9991 - 0.0009 \tag{28}$$

$$= 0.9982 \tag{29}$$

- +1 each for (26), (27), (28), and (29) (no partial credit in any step)

If the student used the properties of symmetric limits problems...

$$P(4.713 \le \overline{X} \le 5.339) = P(-3.12 \le Z \le 3.12) \tag{30}$$

$$= 1 - 2P(Z \le -3.12) \tag{31}$$

$$\approx 1 - 2(0.0009) \tag{32}$$

$$= 0.9982 \tag{33}$$

- +1 each for (30), (31), (32), and (33) (no partial credit in any step)

Note.

- Pay careful attention to whether the student uses the usual method or the properties of symmetric limits problems.
- If the student shows more work than is given here and obtains the correct answer, award full credit.
- Follow any mistake the student makes in (a) above.

14. Professor Spellman is studying the lifespan of nuclear power plants. An analysis of 74 nuclear power plants shows that 23% of them are decommissioned within the first twenty-five years.

(a) Professor Spellman wishes to construct a central 96% confidence interval for the proportion of all nuclear power plants that are decommissioned within the first twenty-five years. Find an appropriate critical number. (1 pt.) The 96% z^* critical number is 2.054. Award one point to any student who gives this as their answer.

(b) Construct a central 96% confidence interval for the proportion of all nuclear power plants that are decommissioned within the first twenty-five years. Round *both* endpoints to *three* decimal places. (5 pts.)

$$\text{CI} = .23 \pm 2.054\sqrt{\frac{.23(1-.23)}{74}} \tag{34}$$

$$= \left(.23 - 2.054\sqrt{\frac{.23(1-.23)}{74}}, .23 + 2.054\sqrt{\frac{.23(1-.23)}{74}}\right) \tag{35}$$

$$= \left(.23 - 2.054\sqrt{\frac{.1771}{74}}, .23 + 2.054\sqrt{\frac{.1771}{74}}\right) \tag{36}$$

$$\approx \left(.23 - 2.054\sqrt{.00239}, .23 + 2.054\sqrt{.00239}\right) \tag{37}$$

$$\approx (.23 - 2.054(.04889), .23 + 2.054(.04889)) \tag{38}$$

$$\approx (.23 - .10042, .23 + .10042) \tag{39}$$

$$\approx (0.130, 0.330) \tag{40}$$

- +1 for point estimate of 0.23
- +1 for numerator of fraction in square root (as shown in (34); no partial credit)
- +1 for denominator of fraction in square root
- +1 for multiplying square root of fraction by z^* critical number
- +1 for (40) with correct rounding

Note.

- Students need not show every calculation given to receive full credit.
- If the student gives (0.330, 0.130) as their answer and shows all work, deduct one point.
- Follow any mistake the student made in (a) above.

(c) Find a conservative estimate for the sample size required to estimate the proportion of all nuclear power plants that are decommissioned within the first twenty-five years to within 0.008 with 96% confidence. (3 pts.)

$$n = \frac{1}{4}\left(\frac{2.054}{.008}\right)^2 \qquad n = .5(1-.5)\left(\frac{2.054}{.008}\right)^2 \tag{41}$$

$$\approx 16480.141 \qquad \approx 16480.141 \tag{42}$$

$$\uparrow 16481 \qquad \uparrow 16481 \tag{43}$$

- +1 for squared fraction in (41) (no partial credit)
- +1 for multiplying squared fraction by one-fourth (either side is acceptable)
- +1 for (43)

15. The Math Department at SUNY Albany is investigating the number of staples used for student exams. The University believes that the Department uses an average of 17936 staples per semester. However, the Department Chair thinks the University's belief is an underestimate. To assess this, the Chair asks Professors Hulbert, Lam, and Lamatina to randomly select 101 semesters and count the number of staples used. Their analysis found an average of 18548 staples used per semester, with a standard deviation of 3058 staples. The Chair wishes to know if there is enough evidence to conclude that the University's belief is an underestimate.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

① H_0: $\mu = 17936$ vs. H_1: $\mu < 17936$

② H_0: $\mu = 17936$ vs. H_1: $\mu > 17936$

③ H_0: $\mu = 17936$ vs. H_1: $\mu \neq 17936$

(b) Compute the t^* test statistic. Round your answer to *one* decimal place. (3 pts.)

(44) $$t^* = \frac{18548 - 17936}{\left(\frac{3058}{\sqrt{101}}\right)}$$

(45) $$\approx \frac{612}{304.28237}$$

(46) $$\approx 2.0^*$$

- +1 for numerator in (44)
- +1 for fraction in denominator of (44) including square root (no partial credit)
- +1 for (46) with correct rounding

Note. Another acceptable answer for the denominator in (45) is 304.28224.

(c) The test statistic has a t distribution with how many degrees of freedom? (1 pt.)

① 102 ② 100 ③ 99 ④ 101

(d) Compute the *p-value* of the test, and express your answer to *three* decimal places. (2 pts.)

(47) $$\alpha_0 = P_{H_0}(T \geq 2.0)$$

(48) $$\approx 0.024$$

- +1 each for (47) and (48)

(e) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject H_0 at $\alpha = 0.05$. There is not enough evidence to conclude that the University's belief is an underestimate.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. There is a slight amount of evidence to conclude that the University's belief is an underestimate.

③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. There is a convincing amount of evidence to conclude that the University's belief is an underestimate.

④ Reject H_0 at $\alpha = 0.001$. There is an overwhelming amount of evidence to conclude that the University's belief is an underestimate.

16. Recently, the State of New York has been exploring ways to reduce its energy expenses. One such initiative, championed by Governor Hochul, involves converting all incandescent light bulbs to LED light bulbs. The manufacturer of a certain brand of LED light bulb asserts that their bulbs are more likely to last at least five years than incandescent ones. Before entering into a supply contract with the manufacturer on behalf of the State, however, Governor Hochul asks Professor Young to conduct an experiment comparing both types of light bulbs. The objective is to determine if the LED light bulbs are more likely to last at least five years than the incandescent ones. Summary statistics from Professor Young's experiment are shown below:

Type of Light Bulb	Incandescent	LED Bulb
Number of Bulbs	12537	12413
Number of Successful Bulbs	6354	6498
Proportion of Successful Bulbs	50.682%	52.348%

In his final report to Governor Hochul, Professor Young wishes to use the 2-Sample Z Test for Equal Proportions to assess whether there is sufficient evidence to conclude that the proportion of all LED light bulbs that are successful is higher than the proportion of all incandescent light bulbs that are successful. Let p_1 represent the proportion of all incandescent light bulbs that are successful, and let p_2 denote the proportion of all LED light bulbs that are successful.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

① (selected) $H_0\colon p_1 = p_2$ vs. $H_1\colon p_1 < p_2$

② $H_0\colon p_1 = p_2$ vs. $H_1\colon p_1 > p_2$

③ $H_0\colon p_1 = p_2$ vs. $H_1\colon p_1 \neq p_2$

(b) Are all requirements for the 2-Sample Z Test for Equal Proportions met? (1 pt.)

① Yes, because both populations have normal distributions.

② Yes, because both sample sizes are sufficiently large.

③ (selected) Yes, because both sets of success-failure condtions are met.

④ No, because none of the above are true.

(c) Compute the pooled sample proportion. Round your answer to *five* decimal places. (3 pts.)

(49) $$\hat{p}_c = \frac{12537(.50682) + 12413(.52348)}{12537 + 12413} \qquad \hat{p}_c = \frac{6354 + 6498}{12537 + 12413}$$

(50) $$\approx 0.51511 \qquad \approx 0.51511$$

- $\boxed{+1}$ for numerator in (49) (either side is acceptable)
- $\boxed{+1}$ for denominator in (49)
- $\boxed{+1}$ for (50) with correct rounding

(d) Compute the z^* test statistic. Round your answer to *two* decimal places, where appropriate. (5 pts.)

(51) $$z^* = \frac{.50682 - .52348}{\sqrt{\frac{.51511(1-.51511)}{12537} + \frac{.51511(1-.51511)}{12413}}} \qquad z^* = \frac{\frac{6354}{12537} - \frac{6498}{12413}}{\sqrt{\frac{.51511(1-.51511)}{12537} + \frac{.51511(1-.51511)}{12413}}}$$

(52) $$\approx -2.63^{**} \qquad \approx -2.63^{**}$$

- $\boxed{+1}$ for numerator in (51) (either side is acceptable; no partial credit)
- $\boxed{+2}$ for fractions in denominator of (51) (one point per fraction; no partial credit)
- $\boxed{+1}$ for taking square root of sum of fractions in denominator of (51)
- $\boxed{+1}$ for (52) with correct rounding

Note.

- Another acceptable answer for (52) is -2.64.
- Follow any mistake the student makes on the previous page.

(e) Compute the *p-value* of the test, and express your answer to *four* decimal places. (3 pts.)

(53) $$\alpha_0 = P_{H_0}(Z \leq -2.63)$$

(54) $$\approx 0.0043$$

- $\boxed{+2}$ for (53) (one point for Z and one point for -2.63)
- $\boxed{+1}$ for (54)

Note.

- If the student obtains -2.64 as their answer in (d) above, award full credit for providing $\alpha_0 \approx 0.0041$ as their answer to this part.
- Follow any mistake the student makes previously.

(f) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject H_0 at $\alpha = 0.05$. Professor Young does not have enough evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. Professor Young has a slight amount of evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.

❸ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. Professor Young has a convincing amount of evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.

④ Reject H_0 at $\alpha = 0.001$. Professor Young has an overwhelming amount of evidence to conclude that the proportion of all LED bulbs that are successful is higher than the proportion of all incandescent bulbs that are successful.

17. In a weekly coordination meeting towards the beginning of the Spring 2025 semester, Professor Medina shared his belief with Professor Lange that student success in AMAT 108 is linked to the number of Brightspace announcements made by instructors. Professor Lange, the Spring 2025 AMAT 108 exam writer, suggested testing this claim during the current semester. They agreed on a study involving two groups of AMAT 108 sections: one with at least eight Brightspace announcements and another with fewer than eight. The data collected were Exam 2 scores out of 100, which leads to the summary statistics below:

# Announcements	< 8	≥ 8
Average	71.615	82.928
Standard Deviation	25.305	15.637
Number of Students	118	120

For the purpose of this problem, assume the scores in both samples are drawn from normally-distributed populations. Professor Lange and Professor Medina wish to use Welch's T Test to see if there is enough evidence to support Professor Medina's claim that significant differences exist in student performance based on the number of Brightspace announcements made. Let μ_1 be the average score in sections with fewer than eight announcements and μ_2 be the average score in sections with at least eight announcements.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

① H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 < \mu_2$

② H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 > \mu_2$

③ (selected) H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$

(b) Are all requirements for Welch's T Test met? (1 pt.)

① (selected) Yes, because both populations have normal distributions.

② Yes, because both samples have normal distributions.

③ Yes, because both sample sizes are sufficiently large.

④ Yes, because the requirements for the One-Sample T Test for μ are met for each sample.

⑤ No, because none of the above are true.

(c) Compute the t^* test statistic. Round your answer to *one* decimal place. (5 pts.)

(55)
$$t^* = \frac{71.615 - 82.928}{\sqrt{\frac{25.305^2}{118} + \frac{15.637^2}{120}}}$$

(56)
$$\approx -4.1^{***}$$

- [+1] for numerator in (55) (no partial credit)
- [+2] for fractions in denominator of (55) (one point per fraction; no partial credit)
- [+1] for taking square root of sum of fractions in denominator of (55)
- [+1] for (56) with correct rounding

(d) The test statistic has a t distribution with how many degrees of freedom? (1 pt.)

① 113

② 114

③ 115

④ 116

⑤ 117

⑥ 118

⑦ 119

⑧ 120

(e) Compute the *p-value* of the test, and express your answer to *three* decimal places. (3 pts.)

(57) $\alpha_0 = 2P_{\mathrm{H}_0}(T \leq -4.1)$

(58) $= 2P_{\mathrm{H}_0}(T \geq 4.1)$

(59) $\approx 2(0)$

(60) $= 0.$

- +1 for recognizing that the area of one of the tails needs to be found (as shown in either (57) or (58))
- +1 for multiplying area by 2 (as shown in (57) and (58))
- +1 for (60)

(f) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject H_0 at $\alpha = 0.05$. There is not enough evidence to conclude that Professor Medina's claim is correct.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. There is a slight amount of evidence to conclude that Professor Medina's claim is correct.

③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. There is a convincing amount of evidence to conclude that Professor Medina's claim is correct.

④ Reject H_0 at $\alpha = 0.001$. There is an overwhelming amount of evidence to conclude that Professor Medina's claim is correct.