UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT 108 Elementary Statistics
Spring 2025

Exam 2
Version 1

Answer Key

Directions: You have **80 minutes** to answer the following questions. ***No notes, textbooks, mobile phones or other aids are allowed. Only scientific calculators are allowed.*** For all multiple-choice questions, select **one** answer from among the choices given. No explanation is required to be shown and no partial credit will be given. Make sure to **completely** fill in the circle corresponding to your chosen answer. For all free-response questions, you **must** show all necessary work to receive full credit. An answer with no work, even if correct, will not receive full credit. Please circle or box your final answer. All work, if needed, is to be rounded to ***five*** decimal places.

Do not detach any pages. Please choose your section with a check mark (✓) in the left-most column.

✓	Instructor Name	Meeting Time	Meeting Days	Meeting Location	Section
	John Habib	11:40AM	M/W	HU 124	1651
	Seth Hulbert	09:00AM	T/TH	SS 116	3998
		12:00PM		HU 124	4046
	Tung Lam	09:00AM	T/TH	FA 126	6998
		01:30PM		TA 118	2761
	James Lamatina	01:10PM	M/W	LC 2	1648
		03:00PM		LC 25	3402
		09:00AM	T/TH	LC 3B	3209
	Chris Lange	03:00PM	T/TH	SS 255	1649
		04:30PM			1653
	Douglas Rosenberg	03:00PM	M/W	BB B006	1652
		04:30PM		HU 133	1654
	Sam Spellman	01:10PM	M/W	FA 126	1650
	Alea Wittig	10:30AM	T/TH	HU 123	3382
		12:00PM		HU 129	3399
	Peter Young	08:00AM	M/W	SS 255	3508
		11:40AM		HU 123	3406

1. Suppose Z has a standard normal distribution. Correct to four decimal places, which of the following is equal to $P(-3.42 \leq Z \leq 2.05)$? (1 pt.)

① 0.9798
② 0.9795
③ 0.9801
④ 0.0003
⑤ None of the previous options.

Questions 2-4 are based on the following. Suppose A and B are events with $P(A) = 0.34$ and $P(B) = 0.28$.

2. Find $P(B^C)$. (1 pt.)

① 0.34
② 0.66964
③ 0.55147
④ 0.72
⑤ 0.62

3. Assume A and B are mutually exclusive (disjoint). Find $P(A \cup B)$. (1 pt.)

① 0.34
② 0.66964
③ 0.55147
④ 0.72
⑤ 0.62

4. Suppose $P(A \cap B) = 0.1875$. Find $P(A|B)$. (1 pt.)

① 0.34
② 0.66964
③ 0.55147
④ 0.72
⑤ 0.62

5. Find the closest value of c so that $P(Z \leq c) \approx 0.0329$. (1 pt.)

① -2.73 ④ -1.84

② -1.83 ⑤ -2.72

③ -1.85

6. Prior to building a nearby bridge, the New York State Thruway Authority asks Professor Young to inspect 846 metal bolts. The inspections Professor Young conducts find that 319 of them fail to meet engineering standards. Use this information to compute all the following probabilities. Round all answers to *five* decimal places, where appropriate.

(a) The first metal bolt Professor Young selects for inspection fails to meet engineering standards. (3 pts.)

(1) $$\frac{319}{846} \approx 0.37707$$

- +1 for numerator in left-hand side of (1)
- +1 for denominator in left-hand side of (1)
- +1 for right-hand side of (1) with correct rounding

(b) The second metal bolt Professor Young selects for inspection fails to meet engineering standards, *given* that the first metal bolt Professor Young selects for inspection meets engineering standards. Assume we are *sampling without replacement*. (3 pts.)

(2) $$\frac{319}{845} \approx 0.37751$$

- +1 for numerator in left-hand side of (2)
- +1 for denominator in left-hand side of (2)
- +1 for right-hand side of (2) with correct rounding

Note. Deduct one point if the student put 318, 527, or 526 as the numerator. Also deduct a point if the student put 846 as the denominator.

(c) The second metal bolt Professor Young selects for inspection fails to meet engineering standards, *given* that the first metal bolt Professor Young selects for inspection fails to meet engineering standards. Assume we are *sampling without replacement*. (2 pts.)

(3) $$\frac{318}{845} \approx 0.37633$$

- +1 for numerator in left-hand side of (3)
- +1 for right-hand side of (3) with correct rounding

Note. Deduct one point if the student put 319, 527, or 526 as the numerator. Also deduct a point if the student put 846 as the denominator. No points should be awarded if both the numerator and denominator are incorrect, even if the calculation was done correctly.

(d) None of the first five metal bolts Professor Young selects for inspection meet engineering standards. Assume we are *sampling with replacement*. (3 pts.)

(4) $$\frac{319}{846} \times \frac{319}{846} \times \frac{319}{846} \times \frac{319}{846} \times \frac{319}{846} = \left(\frac{319}{846}\right)^5 \approx 0.00762$$

If the multiplication of multiple fractions is shown...

- $\boxed{+1}$ for showing the fraction 319/846 five times in (4) (no partial credit)
- $\boxed{+1}$ for multiplying all five fractions together in (4) (no partial credit)
- $\boxed{+1}$ for right-most side of (4) with correct rounding

If a fraction raised to a power is shown...

- $\boxed{+1}$ for showing the fraction 319/846 once (no partial credit)
- $\boxed{+1}$ for raising fraction to the fifth power
- $\boxed{+1}$ for right-most side of (4) with correct rounding

Note.

- Students may also show either the product of their answer in (a) five times or their answer in (a) raised to the fifth power to receive full credit. The decimal form leads to the same answer, correct to five decimal places.
- Follow any mistake made in the calculations or from previous parts.

(e) At least one of the first five metal bolts Professor Young selects for inspection meets engineering standards. Assume we are *sampling with replacement*. (3 pts.)

(5) $$1 - \left(\frac{319}{846}\right)^5 \approx 0.99238$$

- $\boxed{+1}$ for including the $1-$ in left-hand side of (5)
- $\boxed{+1}$ for showing work or answer from (d) (either is acceptable)
- $\boxed{+1}$ for right-hand side of (5) with correct rounding

Note. Follow any mistake made in (d).

7. Suppose X is a discrete random variable with the probability distribution shown below:

X	4	7	9
$P(X)$	0.07	0.12	0.81

(a) Find the mean (expected) value of X. Do *not* round your answer. (3 pts.)

(6) $$\mu = 4(0.07) + 7(0.12) + 9(0.81)$$

(7) $$= 8.41$$

- $\boxed{+1}$ for multiplying in the columns of the table
- $\boxed{+1}$ for adding products together, as shown in (6)
- $\boxed{+1}$ for (7)
- $\boxed{-2}$ if student shows

(8) $$\mu = \frac{4 + 7 + 9}{3}$$

(9) $$\approx 6.667$$

- $\boxed{-3}$ if student shows

(10) $$\mu = \frac{0.07 + 0.12 + 0.81}{3}$$

(11) $$\approx 0.333$$

(b) Find the standard deviation of X. Round your answer to *three* decimal places. (5 pts.) **If the standard method is shown...**

$$\sigma = \sqrt{.07(4-8.41)^2 + .12(7-8.41)^2 + .81(9-8.41)^2} \tag{12}$$

$$\approx 1.372 \tag{13}$$

- $\boxed{+1}$ for squaring all three deviations from the mean
- $\boxed{+1}$ for multiplying squared deviations by associated probability
- $\boxed{+1}$ for adding products together
- $\boxed{+1}$ for taking square root
- $\boxed{+1}$ for (13) with correct rounding

If the simple calculation method is shown...

$$E(X^2) = 4^2(.07) + 7^2(.12) + 9^2(.81) \tag{14}$$

$$= 72.61 \tag{15}$$

$$\sigma = \sqrt{72.61 - 8.41^2} \tag{16}$$

$$\approx 1.372 \tag{17}$$

- $\boxed{+1}$ for (14)
- $\boxed{+1}$ for (15)
- $\boxed{+1}$ for difference between (15) and square of (7)
- $\boxed{+1}$ for taking square root
- $\boxed{+1}$ for (17) with correct rounding

Note. Follow the student's work carefully, and pay close attention to which method the student uses.

8. Suppose Y has a uniform distribution on the interval $(3, 29)$.

(a) Find the height of the density curve. *Leave your answer in fractional form.* (2 pts.)

$$f(y) = \frac{1}{29-3} = \frac{1}{26} \longrightarrow \text{height} = \frac{1}{26} \tag{18}$$

- $\boxed{+1}$ for $29 - 3$ in denominator (this is the student's work shown)
- $\boxed{+1}$ for $1/26$ given as height

Note.

- If the student correctly gives the density function and shows their work but does not specify the height of the density curve, do not deduct any credit.
- If the student expresses the height of the density curve as a decimal, regardless of the rounding, deduct one point.

(b) Compute $P(Y \leq 12)$. Round your answer to *five* decimal places, if needed. (3 pts.)

$$P(Y \leq 12) = \frac{1}{26}(12-3) = \frac{12-3}{26} \tag{19}$$

$$\approx 0.34615 \tag{20}$$

- $\boxed{+1}$ for showing $12 - 3$ as width of rectangle in either form shown in (19)
- $\boxed{+1}$ for showing 26 as height of rectangle in either form shown in (19)
- $\boxed{+1}$ for (20) with correct rounding

Note. If the student computes $12/26 \approx 0.46154$, deduct one point.

9. Suppose that the probability that an individual walks into a sporting goods store is 0.328. Also suppose that the probability that an individual walks into a sporting goods store and buys sporting safety equipment is 0.197. Compute the probability that an individual buys sporting safety equipment, given that the individual walks into a sporting goods store. Round your answer to *five* decimal places. (3 pts.)

(21) $$\frac{0.197}{0.328} \approx 0.60061$$

- +1 for numerator in left-hand side of (21)
- +1 for denominator in left-hand side of (21)
- +1 for right-hand side of (21) with correct rounding

10. A recently-published statistical study claims that 41% of all Americans are unknowingly at high risk of a heart attack or stroke. Professor Medina selects 4612 Americans at random and assesses their risk of having a heart attack or stroke.

(a) Is the distribution of $\hat{p}$ normal? (1 pt.)

① Yes, because the population of interest has a normal distribution.

② No, but $\hat{p}$ has an approximate normal distribution because the sample size is large enough.

❸ No, but $\hat{p}$ has an approximate normal distribution because the success-failure conditions are met.

④ No, and we cannot conclude anything about the distribution of $\hat{p}$.

(b) Find the mean and standard deviation of the distribution of $\hat{p}$. Round the standard deviation to *five* decimal places. (4 pts.)

(22) $$\mu = 0.41$$

(23) $$\sigma = \sqrt{\frac{.41(1-.41)}{4612}}$$

(24) $$\approx 0.00724$$

- +1 for (22)
- +1 for fraction in (23) (no partial credit)
- +1 for taking square root of fraction in (23)
- +1 for (24) with correct rounding

(c) Professor Medina finds that 42.98% of the 4612 individuals he selected are unknowingly at high risk of a heart attack or stroke. Find the z-score for this sample proportion. Round your answer to *two* decimal places. (3 pts.)

(25) $$z = \frac{0.4298 - 0.41}{0.00724}$$

(26) $$\approx 2.73$$

- +2 for (25) (one point for numerator and one point for denominator; no partial credit; see note below)
- +1 for (26) with correct rounding

Note. Follow any mistakes the student made in (b).

(d) Use your answer to (c) to compute the probability that no less than 42.98% of the 4612 individuals Professor Medina selected are unknowingly at high risk of a heart attack or stroke. *Hint:* Find $P(\hat{p} \geq 0.4298)$. (2 pts.) **If the student uses the probability complement rule...**

(27) $$P(\hat{p} \geq 0.4298) \approx P(Z \geq 2.73)$$

(28) $$= 1 - P(Z \leq 2.73)$$

(29) $$\approx 1 - 0.9968$$

(30) $$= 0.0032$$

- $\boxed{+1}$ for (28)
- $\boxed{+1}$ for (30)

If the student uses the symmetry of the standard normal distribution density curve...

(31) $$P(\hat{p} \geq 0.4298) \approx P(Z \geq 2.73)$$

(32) $$= P(Z \leq -2.73)$$

(33) $$= 0.0032$$

- $\boxed{+1}$ for (32)
- $\boxed{+1}$ for (33)

11. Professor Lange claims that, in the month of July, his average 18-hole golf score is 74.5 with a standard deviation of 2.934. Professor Lamatina, thinking that Professor Lange is being facetious as usual, randomly selects 320 of Professor Lange's 18-hole golf scores from the month of July over several calendar years. For the purposes of this problem, assume the distribution of all of Professor Lange's golf scores over the selected calendar years is normal.

(a) Is the distribution of $\overline{X}$ normal? (1 pt.)

● ① Yes, because the population has a normal distribution with mean 74.5 and standard deviation 2.934.

② No, but the distribution is approximately normal because the sample size is large enough.

③ No, and we cannot conclude anything about the distribution of $\overline{X}$.

(b) Find the mean and standard deviation of the distribution of $\overline{X}$. Round the standard deviation to *five* decimal places. (4 pts.)

(34) $$\mu = 74.5$$

(35) $$\sigma = \frac{2.934}{\sqrt{320}}$$

(36) $$\approx 0.16402$$

- $\boxed{+1}$ for (34)
- $\boxed{+1}$ for numerator in (35)
- $\boxed{+1}$ for denominator in (35) including square root (no partial credit)
- $\boxed{+1}$ for (36) with correct rounding

(c) The sample of Professor Lange's golf scores that Professor Lamatina selects is found to have an average of 74.125, correct to three decimal places. Find the z-score for this sample mean. Round your answer to *two* decimal places. (3 pts.)

$$z = \frac{74.125 - 74.5}{0.16402} \tag{37}$$

$$\approx -2.29 \tag{38}$$

- $\boxed{+1}$ for numerator in (37) (no partial credit; see note below)
- $\boxed{+1}$ for denominator in (37) (see note below)
- $\boxed{+1}$ for (38) with correct rounding

Note. Follow any mistakes the student made in (b).

(d) Use your answer in (c) to compute the probability that the average of the 320 scores Professor Lamatina selected is no more than 74.125. *Hint:* Find $P(\overline{X} \leq 74.125)$. (1 pt.)

$$P(\overline{X} \leq 74.125) \approx P(Z \leq -2.29) \tag{39}$$

$$\approx 0.0110 \tag{40}$$

- $\boxed{+1}$ for (40)

Note. If the student made any mistakes in (b) or (c), follow it through.

(e) Compute $P(74.125 \leq \overline{X} \leq 74.875)$. *Hint:* This is an example of a symmetric-limits problem. (4 pts.) **If the student used the standard method of computing standard normal probabilities. . .**

$$P(74.125 \leq \overline{X} \leq 74.875) \approx P\left(-2.29 \leq Z \leq \frac{74.875 - 74.5}{0.16402}\right) \tag{41}$$

$$\approx P(-2.29 \leq Z \leq 2.29) \tag{42}$$

$$= P(Z \leq 2.29) - P(Z \leq -2.29) \tag{43}$$

$$\approx 0.9890 - 0.0110 \tag{44}$$

$$= 0.9780 \tag{45}$$

- $\boxed{+1}$ for (42)
- $\boxed{+1}$ for (43)
- $\boxed{+1}$ for finding 0.9890 from Z table and using answer from (d) for $P(Z \leq -2.29)$ (no partial credit)
- $\boxed{+1}$ for (45)

If the student used the properties of symmetric-limits problems. . .

$$P(74.125 \leq \overline{X} \leq 74.875) = P(-2.29 \leq Z \leq 2.29) \tag{46}$$

$$= 1 - 2P(Z \leq -2.29) \tag{47}$$

$$\approx 1 - 2(0.0110) \tag{48}$$

$$= 0.9870 \tag{49}$$

- $\boxed{+1}$ each for (46), (47), (48), and (49)

Note. While the intent was for students to use (46) - (49) to answer this question, either method is acceptable. Pay careful attention to which method the student uses and award the points accordingly. Also, follow any mistakes made previously.