Practice Assessment 9

Improper Integrals

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$$

If the limit above exits, we say that the improper integral **converges**. Otherwise, it is said to **diverge**.

1. Determine whether the following improper integrals converge or diverge. If the integral converges, then compute its value.

   (a) $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{x}{\sqrt{x^2+1}}\,dx$

   (c) $\displaystyle\int_{-\infty}^2 e^{2x}\,dx$

The improper integral $\int_{-\infty}^{\infty} f(x)\,dx$ is defined by

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx.$$

The integral on the left is said to converge only when **both** integrals on the right converge. Otherwise it is said to diverge.

2. Determine whether the following improper integrals converge or diverge. If the integral converges, then compute its value.

   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{4+x^2}\,dx$

   (b) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx$

Comparison Test: Suppose f and g are continuous on $[a, \infty)$ with $0 \leq f(x) \leq g(x)$ for all $x \geq a$. Then

(a) If $\int_a^\infty g(x)\,dx$ converges, so does $\int_a^\infty f(x)\,dx$

(b) If $\int_a^\infty f(x)\,dx$ diverges, so does $\int_a^\infty g(x)\,dx$

3. Use the Comparison Test to determine whether the improper integral converges or diverges.

   (a) $\displaystyle\int_1^\infty e^{-(1+x^2)}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{1}{\sqrt{1+x^5}}\,dx$

   (c) $\displaystyle\int_1^\infty \frac{e^x}{\sqrt{1+x^2}}\,dx$

   (d) $\displaystyle\int_1^\infty \frac{1}{\sqrt{e^{x^2}+x+\cos(x)}}\,dx$