Practice Assessment 7

Trigonometric Substitutions

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

The following trigonometric substitutions are used to handle integrals involving radicals:

$$\sqrt{a^2+x^2} \quad \text{requires} \quad x = a\tan\theta$$

$$\sqrt{a^2-x^2} \quad \text{requires} \quad x = a\sin\theta$$

$$\sqrt{x^2-a^2} \quad \text{requires} \quad x = a\sec\theta$$

1. Compute the following integrals:

   (a) $\displaystyle\int \frac{1}{\sqrt{x^2+1}}\,dx$

   (b) $\displaystyle\int \sqrt{x^2+4}\,dx$

2. Compute the following integrals:

(a) $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

(b) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$

(c) $\displaystyle\int_{-1}^{2} \sqrt{4-x^2}\,dx.$