Practice Assessment 19

Parametric Equations

Parametric Equations: If x and y are continuous functions of t on an interval I, then the equations

$$x = x(t) \quad \text{and} \quad y = y(t), \quad t \in I,$$

are called the **parametric equations** and t is call the **parameter**. The set of points (x, y) obtained as t varies over the interval I is called a **parametric curve** or plane curve, and is denoted by C.

1. Sketch the curves below by eliminating the parameter t and find an equation in $xy-$coordinates whose graph contains the given curve.

   (a) $x(t) = t, \quad y(t) = t + 6$

   (b) $x(t) = t^2 + 2t, \quad y(t) = t + 1$

   (c) $x(t) = e^t, \quad y(t) = e^{2t}$

   (d) $x(t) = \sin(t), \quad y(t) = \cos^2(t)$

2. (a) Find the parametric equations for the graph $y + \sqrt{x} = e^{2x}$.

   (b) Find the parametric equations for the line containing the point $P_0 = (2, 4)$ and with slope 3.

   (c) Find parametric equations for the ellipse $\dfrac{x^2}{7} + \dfrac{y^2}{13} = 1$.

3. A particle moves in the plane so that at time t its position is given by $x(t) = t + 4, y(t) = 8 - t^2$. A second particle moves in the plane so that at time t its position is given by $x(t) = t + 4, y(t) = t + 6$.

   (a) Find equations in $xy-$coordinates for each of the curves.

   (b) Do the paths of the particles cross? If so, where?

   (c) Do the particles collide? If so, where and at what time?