Practice Assessment 18

Taylor and Maclaurin Series

Taylor and Maclaurin Series: If f has derivatives of all orders at $x = a$, then the **Taylor Series** for the function f at a is

$$\sum_{k=0}^{\infty} \frac{f(k)(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots.$$

The Taylor series for f at $a = 0$ is known as the **Maclaurin** series for f.

1. Find the Taylor or Maclaurin series for the given function expanded about the given point and determine the numbers x for which the series converges.

   (a) $f(x) = e^{2x}, \qquad a = 0$

   (b) $f(x) = 1 + x^2, \qquad a = 2$

   (c) $f(x) = \ln(3 + x), \qquad a = 0$

   (d) $f(x) = \sqrt{x}, \qquad a = 4$

Taylor Polynomial: If f has n derivatives at $x = a$, then the n-th **Taylor Polynomial** for the function f at $x = a$ is

$$p_n(x) = \sum_{k=0}^{n} \frac{f(k)(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f(n)(a)}{n!}(x-a)^n.$$

The n-th Taylor polynomial for f at $a = 0$ is known as the n-th **Maclaurin Polynomial** for f.

2. Find the n-th Taylor Polynomial for the function f expanded about $x = a$.

   (a) $f(x) = e^{-x}, \qquad a = 0, \qquad n = 4$

   (b) $f(x) = \cos(x), \qquad a = \pi/4, \qquad n = 6$

   (c) $f(x) = \dfrac{1}{1+x^2}, \qquad a = 0, \qquad n = 2$

   (d) $f(x) = \sqrt{1-x}, \qquad a = 0, \qquad n = 3$