Practice Assessment 16
Power Series and Functions

A **power series** centered at the $x = a$ is an expression of the form

$$\sum_{k=0}^{\infty} c_k(x-a)^k = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

where the coefficients $c_0, c_1, c_2, \dots$ are constants and x is regarded as the independent variable.

1. Find the interval of convergence of the given power series.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k+2}$

   (b) $\displaystyle\sum_{k=2}^{\infty} \frac{x^k}{\ln(k)}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k!}(x-3)^k$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{k(x-2)^k}{e^k}$

(e) $\displaystyle\sum_{k=2}^{\infty} \frac{(k(7x+1)^k}{2^k}$

2. Given that
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad , \text{for } x \in (-1, 1).$$
Find the power series for each function centered at $a = 0$ and state its radius of convergence.

(a) $f(x) = \dfrac{1}{1-2x}$

(b) $f(x) = \dfrac{1}{1+4x^2}$

(c) $f(x) = \dfrac{x}{1+x^2}$

(d) $f(x) = \dfrac{x-1}{x+1}$