Practice Assessment 15
The Ratio and Root Tests

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

The Ratio Test: Let $a_k > 0$ for all $k = 1, 2, \ldots$ and let

$$\rho = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}.$$

Then, provided the limit exits,

(i) If $\rho < 1$, the series $\sum_{k=1}^{\infty} a_k$ converges.

(ii) If $\rho > 1$, the series $\sum_{k=1}^{\infty} a_k$ diverges.

(iii) If $\rho = 1$, no conclusion may be drawn.

1. Use the Ratio Test to determine whether the given infinite series converges or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{3^k}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{(k+1)!}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{k^k}$

The Root Test: Let $a_k > 0$ for all $k = 1, 2, \ldots$ and let

$$\rho = \lim_{k\to\infty} \sqrt[k]{a_k}.$$

Then, provided the limit exits,

(i) If $\rho < 1$, the series $\sum_{k=1}^{\infty} a_k$ converges.

(ii) If $\rho > 1$, the series $\sum_{k=1}^{\infty} a_k$ diverges.

(iii) If $\rho = 1$, no conclusion may be drawn.

2. Use the Root Test to determine whether the given infinite series converges or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k^k}$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{2k+1}\right)^k$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3 2^{k+3}}{2^{2k}}$

3. Find those numbers $x > 0$ for which the given series converges.

   (a) $\sum_{k=1}^{\infty} \frac{x^{2k}}{2k}$

   (b) $\sum_{k=1}^{\infty} \frac{x^k}{k!}$