# Practice Assessment 14
## Alternating Series

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

**Absolute Convergence**: The series $\sum a_k$ is said to **converge absolutely** if the series of absolute values, $\sum |a_k|$, converges.

1. (a) Show that the following series converges absolutely:

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k^2+1}$$

(b) Show that the following series does not converge absolutely:

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k+1}$$

**Absolute Convergence implies Convergence!** If the series $\sum |a_k|$ converges then so does $\sum a_k$. That is, every absolutely convergent series converges.

2. (a) Show that the series

$$\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{(k+1)!}$$

converges, by applying the Ratio Test to the series

$$\sum_{k=1}^{\infty} \left| \frac{(-1)^k k^2}{(k+1)!} \right|.$$

(b) Demonstrate that the series

$$\sum_{k=1}^{\infty} \frac{\cos(\pi k)}{2^k}$$

converges, by showing that it converges absolutely.

**Alternating Series:** An **alternating series** is a series of the form

$$\sum_{k=0}^{\infty}(-1)^k a_k = a_0 - a_1 + a_2 - a_3 + a_4 - \dots$$

where $a_k > 0$ for each $k = 0, 1, 2, 3, \dots$.

The following theorem indicates the conditions under which an alternating series converges.

**Alternating Series Test:** Let $a_k > 0$ for each $k = 0, 1, 2, 3, \dots$. The alternating series

$$\sum_{k=0}^{\infty}(-1)^k a_k$$

converges if both the following hold:

(i) The sequence $\{a_k\}_{k=0}^{\infty}$ is decreasing, and

(ii) $\lim_{k\to\infty} a_k = 0$.

3. (a) Show that the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{k^2+1}$$

**does not** converge absolutely, by applying the Integral Test to the series

$$\sum_{k=1}^{\infty} \left| \frac{(-1)^{k+1}k}{k^2+1} \right|.$$

(b) Demonstrate that the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{k^2+1}$$

converges, by using the Alternating Series Test.

**Conditional Convergence:** The series $\sum a_k$ **converges conditionally** if $\sum a_k$ converges and $\sum |a_k|$ diverges.

4. Determine whether the series converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{2k+1}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{(k+2)!}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k\sqrt{k}}{k^2+5}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k!}{6^k}$