Practice Assessment 13
The Comparison Tests

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

The Basic Comparison Test: Let $\sum a_k$ and $\sum b_k$ be infinite series with $0 < a_k \leq b_k$ for each $k = 1, 2, \ldots$. Then

(i) If $\sum b_k$ converges, then so does $\sum a_k$.

(ii) If $\sum a_k$ diverges, then so does $\sum b_k$.

1. Use the Basic Comparison Test to determine whether the given infinite series converges or diverges.

   (a) $\displaystyle\sum_{k=1}^{\infty}(k-1)e^{-k}$

   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2+1}$

   (c) $\displaystyle\sum_{k=1}^{\infty}\frac{\sqrt{k}}{k^3+1}$

   (d) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^{1/2}+k^{3/2}}$

The Limit Comparison Test: Let $\sum a_k$ and $\sum b_k$ be infinite series with a_k 0, $b_k > 0$ for all $k = 1, 2, \ldots$. If the limit

$$\rho = \lim_{k\to\infty} \frac{a_k}{b_k}$$

exists, and $\rho \neq 0$, then either both series converge or both series diverge.

2. Use the Limit Comparison Test to determine whether the given infinite series converges or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{k+3}{2k^2+1}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k+\sqrt{k}}{k+k^3}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2-4}{k^3+k+5}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{2k+2}{\sqrt{k^3+2}}$