Practice Assessment 11

Infinite Series

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

An **infinite series** is an expression of the form

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \dots.$$

The infinite series

$$\sum_{k=1}^{\infty} a_k$$

is said to **converge** to the **sum** S if

$$S = \lim_{n\to\infty} S_n,$$

where S_n denotes the n-*th* **partial sum**

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{k=1}^{n} a_k.$$

If the limit S does not exist, the series $\sum_{k=1}^{\infty} a_k$ is said to **diverge**.

1. Write the given repeating decimal as an infinite series.

   (a) $0.66\bar{6}$

   (b) $0.92929\bar{2}$

A useful tool to determine if a series diverges is the **Divergence Test**. It states that if $\lim_{n\to\infty} a_n = c \neq 0$ or $\lim_{n\to\infty} a_n$ does not exists, then the series $\sum_{k=1}^{\infty} a_k$ diverges.

2. Determine whether the given infinite series converges or diverges. If it converges, find its sum.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{3 \cdot 2^k + 3^k}{5^k}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k+2}$

(d) $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

(e) $\sum_{k=1}^{\infty} \cos(\pi k)$

(f) $\sum_{k=4}^{\infty} \frac{1}{k^2-1}$