Practice Assessment 10

Sequences

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

Properties of Limits of Sequences: If $\lim_{n\to\infty} a_n = A$, $\lim_{n\to\infty} b_n = B$, and c is any real number, then

1. $\lim_{n\to\infty} (a_n + b_n) = A + B$
2. $\lim_{n\to\infty} (ca_n) = cA$
3. $\lim_{n\to\infty} (a_n b_n) = AB$
4. $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}, \quad b_n \neq 0, \quad B \neq 0.$

1. Determine whether the following the sequences converge or diverge. If the sequence converges, then compute its limit.

   (a) $\left\{ \dfrac{6n^3 + 5n^2 + 7}{4n^3 - 2n + 2} \right\}$

   (b) $\left\{ \ln(n+4) - \dfrac{1}{2}\ln(n) \right\}$

   (c) $\left\{ (-1)^n \left(\dfrac{n+1}{n} \right) \right\}$

(d) $\left\{\dfrac{\sqrt{2n^2+1}}{n}\right\}$

(e) $\left\{\dfrac{e^n - e^{-n}}{e^n + e^{-n}}\right\}$

(f) $\left\{\sqrt{n+1} - \sqrt{n}\right\}$

Suppose that $\lim_{n\to\infty} a_n = A$ and each number a_n lies in the domain of the function f. If f is continuous at $x = A$, then $\lim_{n\to\infty} f(a_n) = f(A)$.

2. Compute the following limits.

(a) $\lim_{n\to\infty} \tan\left(\dfrac{\pi n^2 + 1}{3 - 4n^2}\right)$.

(b) $\lim_{n\to\infty} \ln\left(\dfrac{n^2 + 1}{(n+2)(n+3)}\right)$.

Squeeze Theorem for Sequences: Consider sequences $\{a_n\}, \{b_n\}$, and $\{c_n\}$. Suppose there is an integer N such that

$$a_n \leq b_n \leq c_n \quad \text{for all} \quad n \geq N.$$

if there is a number L such that

$$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n,$$

then $\{b_n\}$ converges and $\lim_{n\to\infty} b_n = L$.

3. Determine the limit of the following sequences, if it exists, using the Squeeze Theorem. Justify your answers.

(a) $\left\{\dfrac{\cos(n)+\sin(n)}{n+1}\right\}$

(b) $\left\{\dfrac{2^n+3^n}{4^n}\right\}$

(c) $\left\{\dfrac{\sin(n^2)}{n}\right\}$

(d) $\left\{\dfrac{(-1)^n+\cos(n)}{n^2+1}\right\}$

(e) $\left\{\dfrac{n!}{n^n}\right\}$