

AMAT 108

ELEMENTARY STATISTICS
EXAM REVIEW PACKETPrint Name: **Answer Key**

UAlbany Email:

Directions: There are two main ways you might want to write up your work. You may either write on this PDF using a tablet, or you may print this and write in the space provided. **You should practice showing all work.** On the Final Exam, you may receive zero or reduced points for insufficient work. **Your work should be neatly organized and written.** You may receive zero or reduced points for incoherent work.

For each multiple choice question, fill in the circle representing your answer. **For each free-response question,** box or circle your final answer.

These problems are designed to be done with a scientific calculator only. While nothing is stopping you from using a graphing calculator when working through this packet, be aware of the fact that you are not permitted to use graphing calculators on exams so you might want to practice without one.

UNIT 1: Chapters 1-4 of Peck, Short, and Olsen

Questions 1-3 refer to the following scenario. Dr. Medina asks 192 UAlbany faculty members about the size of their home offices. From the responses, it is found that the home offices average 250 square feet in size.

1. Which of the following is the sample?

- ① All UAlbany faculty members. ② The 192 UAlbany faculty members.
③ The size of the home offices. ④ The home offices themselves.

2. Which of the following is the population of interest?

- ① All UAlbany faculty members. ② The 192 UAlbany faculty members.
③ The size of the home offices. ④ The home offices themselves.

3. Which of the following correctly describes the variable of interest?

- ① It is a numerical variable that gives discrete data. ② It is a categorical variable on four categories.
③ It is a categorical variable on seven categories. ④ It is a numerical variable that gives continuous data.

4. All of the following are valid as a relative frequency except...

- ① 0.4 ② -0.2 ③ 1.0 ④ 0.7

5. Suppose X is a categorical variable on three categories. Dr. Medina selects 195 observations on X at random. Which of the following is not an absolute frequency he can expect to see in any category?

- ① 75 ② 200
③ 115 ④ None of the above.

Questions 6 and 7 are based on the following. Mr. Lange surveys 639 professional athletes on the amount of time they spend training. The survey finds that 62% train for more than four hours each day.

6. The population of interest in this study is...

- ① ... all professional athletes. ② ... the 639 professional athletes.
③ ... 62% ④ ... 38%

7. The sample in this study is...

- ① ... all professional athletes. ② ... the 639 professional athletes.
③ ... 62% ④ ... 38%

For Questions 8-16, determine whether the statement given is true or false.

8. The entire collection of individuals or objects about which information is desired is called the population of interest.

- ① True ② False

9. Methods for summarizing data make up the branch of statistics called descriptive statistics.

- ① True ② False

10. A primary use of inferential statistics is to make generalizations from a sample to a population.

- ① True ② False

11. Frequency distributions can be used with any kind of data.

- ① True ② False

12. The relative frequency for a particular category is the number of times the category appears in the data, relative to the size of the data set.

- ① True ② False

13. Bar charts can be used with any kind of data.

- ① True ② False

14. A data set is discrete if the possible values form entire intervals on the real number line.

- ① True ② False

15. A data set consisting of many observations of a single characteristic is a numerical data set.

- ① True ② False

16. A data set is bivariate if it consists only of numeric variables.

- ① True ② False

17. Dr. Medina is interested in studying the failure rate of a certain kind of car engine part. Which of the following sampling methods leads to the least amount of sampling bias?

- ① He chooses the most recent 371 cars that he sees. ② He selects every car.
③ He uses cluster sampling to choose 294 cars. ④ They will all give an equally representative sample.

18. Sample selection bias is also known as...

- ① ...overcoverage. ② ...undercoverage.
③ ...Both ① and ② are true. ④ None of the above are true.

19. Which of the following is an example of bias in sampling?

- ① A survey on office professionals' software preferences fails to include any accountants.
② Mr. Lange's pedometer tells him he walked 6294 steps when he actually walked 6285 steps.
③ 78% of individuals selected for a phone-in political poll fail to answer the phone.
④ All are examples of bias in sampling.

20. All of the following are proper ways to sample from a population of interest except...

- ① ...voluntary response sampling. ② ...cluster sampling.
③ ...simple random sampling. ④ ...stratified random sampling.

For Questions 21-24, determine whether the statement given is true or false.

21. As given in the definition, a simple random sample of size n is any sample that is selected so that every individual in the population is guaranteed an equal chance of being selected.

- ① True ② False

22. Response bias can occur when responses are not obtained from every individual selected for inclusion in the sample.

- ① True ② False

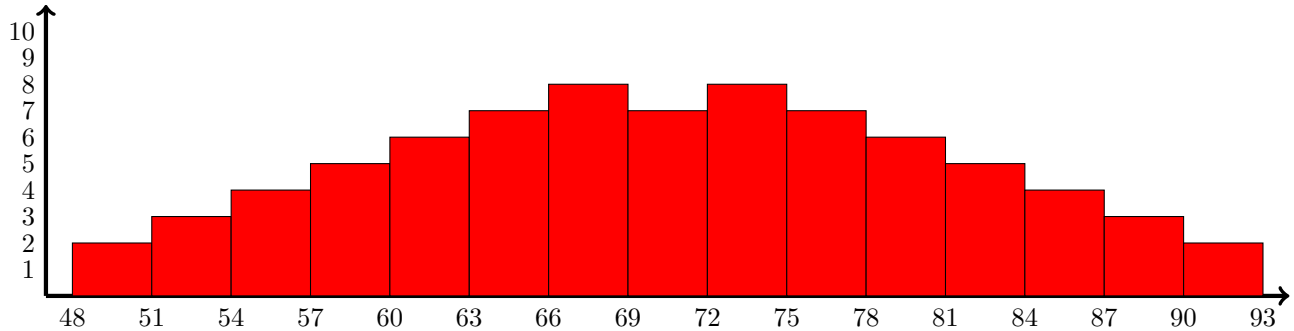
23. Stratified sampling is a sampling method that does not involve simple random sampling.

- ① True ② False

24. Clusters are non-overlapping subgroups of a population that have been identified as homogeneous.

- ☐ True ☒ False

25. The histogram below is of the total number of points in a number of professional American football games.



Select *all* terms that can be used to describe the data set.

- ① unimodal ② bimodal ③ multimodal
④ negatively skewed ⑤ symmetrical ⑥ positively skewed
⑦ uniform ⑧ bell-shaped ⑨ None of these apply.

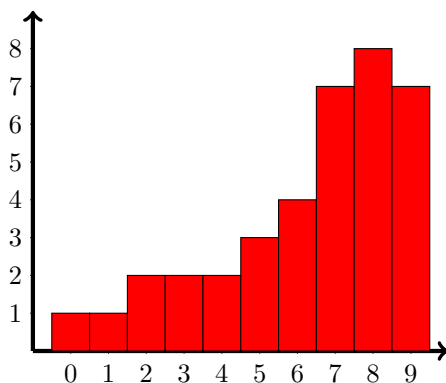
26. Construct a stem-and-leaf plot for the data below. Include a key.

73 71 70 72 80 66 74 78 67 79 75 48

Key: $6|6 = 66$

4	8
5	
6	67
7	01234589
8	0

27. The histogram below is of the number of goals scored against a soccer (international football) team over several years. Select *all* terms that can be used to describe the data set.



- ① unimodal ② bimodal
③ multimodal ④ negatively skewed
⑤ symmetrical ⑥ positively skewed
⑦ uniform ⑧ bell-shaped

28. Any observation that is not unusually small or large is an outlier.

29. A pie chart is most useful for categorical data.

30. One advantage of histograms is that they may be used for large data sets.

31. If the upper tail (or right-hand tail) of a distribution stretches out farther than the lower tail (or left-hand tail), the distribution is positively skewed.

32. A random sample of size 11 was selected from its population of interest. The observations are below:

Compute all of the following and round all answers to *three* decimal places, where appropriate:

$$(2) \quad \approx 77.182$$

73	74	75	75	76	77	78	79	79	81	82
min = 73		LQ = 75		med = 77		UQ = 79		max = 82		

$$(6) \quad \approx 2.892$$

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33. Suppose a data set with 52 observations is found to have a mean of 84 and a standard deviation of 2.

(a) Assume that the data is at least well-approximated by a normal curve. Then approximately what percentage of observations are between 80 and 86? 80 is two standard deviations below the mean of 84, and 86 is one standard deviation above the mean. Recall that 95% of all observations are within two standard deviations of the mean (on both sides), and 68% of all observations are within one standard deviation of the mean (also on both sides). Dividing these percentages in half (by symmetry of all normal curves) and adding them together yields our desired result, 81.5%.

(b) Suppose one of the observations has a value of 81.25. How many standard deviations below the mean is this observation?

$$(7) \quad z = \frac{81.25 - 84}{2}$$

$$(8) \quad = -1.375$$

(c) Suppose another observation has a value of 87.75. How many standard deviations above the mean is this observation?

$$(9) \quad z = \frac{87.75 - 84}{2}$$

$$(10) \quad = 1.875$$

34. The sample median and IQR have the advantage over the sample mean and sample standard deviation for which reason?

① The sample median and IQR are sensitive to outliers.

② The sample median and IQR are not sensitive (robust) to outliers.

③ Both ① and ② are true.

④ None of the above are true.

35. A sample of five observations is given below:

7 6 11 12 9

(a) Compute the sample mean.

$$(11) \quad \bar{x} = \frac{7 + 6 + 11 + 12 + 9}{5}$$

$$(12) \quad = 9$$

(b) Compute the sample variance.

$$(13) \quad s^2 = \frac{(7-9)^2 + (6-9)^2 + (11-9)^2 + (12-9)^2 + (9-9)^2}{4}$$

$$(14) \quad = 6.5$$

(c) Compute the sample standard deviation. Round your answer to *three* decimal places.

$$(15) \quad s = \sqrt{6.5}$$

$$(16) \quad \approx 2.550$$

36. A sample of 11 observations is given below:

76 67 79 80 78 77 89 77 78 76 75

(a) Using *any* method you like, find the median of the data set.

67 75 76 76 77 77 78 78 79 80 89

The median is 77.

(b) Complete the table below:

Lower Fence	Lower Quartile	Median	Upper Quartile	Upper Fence
71.5	76	77	79	83.5

$$\text{IQR} = 79 - 76 = 3$$

(c) Does the data set contain any outliers? If it does, which observations are outliers? **Yes, the minimum and maximum observations are both outliers.**

37. Mr. Lange is interested in studying the cost of a certain kind of onion. To do this, he samples from various grocery stores throughout New York State. The sample has a mean of 8.32 units and a standard deviation of 1.21 units.

(a) Assume that the sample is well-approximated by a normal curve. About what percentage of grocery stores charge no more than 10.74 units for these onions? **10.74 units is two standard deviations above the mean. Recall that 95% of all observations are within two standard deviations of the mean (on both sides). So, 5% of all observations are more than two standard deviations from the mean, and symmetry tells us that 2.5% of all observations are more than two standard deviations above the mean (above 10.74 units, in this case).**

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- (b) Suppose one such grocery store sells these onions for 7 units. How many standard deviations from the mean is this observation? Round your answer to *three* decimal places.

$$(17) \quad z = \frac{7 - 8.32}{1.21}$$

$$(18) \quad \approx -1.091$$

- (c) Suppose another such grocery store sells these onions for 10 units. How many standard deviations from the mean is this observation? Round your answer to *three* decimal places.

$$(19) \quad z = \frac{10 - 8.32}{1.21}$$

$$(20) \quad \approx 1.388$$

- (d) Using your answers to (b) and (c), is 7 units smaller than or larger than $\bar{x}$?

① Smaller than $\bar{x}$

② Larger than $\bar{x}$

- (e) Using your answers to (b) and (c), is 10 units smaller than or larger than $\bar{x}$?

① Smaller than $\bar{x}$

② Larger than $\bar{x}$

38. Which of the following is a characteristic of the population of interest?

① s^2

② μ

③ $\bar{x}$

④ $\hat{p}$

39. A sample of five observations is given below:

3

6

2

8

9

- (a) Compute the sample mean.

$$(21) \quad \bar{x} = \frac{3 + 6 + 2 + 8 + 9}{5}$$

$$(22) \quad = 5.6$$

- (b) Compute the sample variance.

$$(23) \quad s^2 = \frac{(3 - 5.6)^2 + (6 - 5.6)^2 + (2 - 5.6)^2 + (8 - 5.6)^2 + (9 - 5.6)^2}{4}$$

$$(24) \quad = 9.3$$

- (c) Compute the sample standard deviation. Round your answer to *three* decimal places.

$$(25) \quad s = \sqrt{9.3}$$

$$(26) \quad \approx 3.050$$

40. All of the following are population characteristics except...

① σ

② p

③ $\hat{p}$

④ μ

UNIT 2: Chapters 6-8 of Peck, Short, and Olsen

41. Suppose two fair coins are spun, and let S be the sample space. Express S in set notation.

(27) $S = \{HH, HT, TH, TT\}$

42. Let A , B , and C be mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.1$, and $P(C) = 0.4$. Find all of the following:

(a) $P(A \cup B)$.

(28) $P(A \cup B) = P(A) + P(B)$

(29) $= 0.2 + 0.1$

(30) $= 30\%$

(b) $P((A \cap B) \cup B)$. *Hint:* How many outcomes are favorable to both A and B ?

(31) $P((A \cap B) \cup B) = P(\emptyset \cup B)$

(32) $= P(B)$

(33) $= 10\%$

(c) $P((A \cap B) \cup (B \cap C))$. *Hint:* The hint to the previous question will help you with this one.

(34) $P((A \cap B) \cup (B \cap C)) = P(\emptyset \cup \emptyset)$

(35) $= P(\emptyset)$

(36) $= 0\%$

(d) $P(C^C)$

(37) $P(C^C) = 1 - P(C)$

(38) $= 1 - 0.4$

(39) $= 60\%$

(e) $P((A \cup B \cup C)^C)$

(40) $P((A \cup B \cup C)^C) = 1 - P(A \cup B \cup C)$

(41) $= 1 - P(A) - P(B) - P(C)$

(42) $= 1 - 0.2 - 0.1 - 0.4$

(43) $= 30\%$

(f) Based on your answer to the previous question, does the sample space equal the union of all four events?

① Yes

② No

Questions 43 and 44 refer to the following. Suppose E and F are events such that $P(E) = 50\%$, $P(F) = 40\%$, and $P(E \cap F) = 20\%$.

43. Compute $P(E|F)$

$$(44) \qquad P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$(45) \qquad = \frac{0.2}{0.4}$$

$$(46) \qquad = 50\%$$

44. Using your answer to the previous question, are E and F independent?

☒ Yes

☐ No

45. Suppose events A and B are mutually exclusive. Which of the following is true?

☐ $P(A \cap B) = P(A)P(B)$

☐ $P(A \cup B) = P(A)P(B)$

☐ $P(A \cap B) = P(A) + P(B)$

☒ $P(A \cup B) = P(A) + P(B)$

46. Suppose events A and B are independent. Which of the following is true?

☒ $P(A \cap B) = P(A)P(B)$

☐ $P(A \cup B) = P(A)P(B)$

☐ $P(A \cap B) = P(A) + P(B)$

☐ $P(A \cup B) = P(A) + P(B)$

47. Suppose A and B are disjoint events, let S be the sample space, and suppose $S = A \cup B$. Which of the following is true?

☐ $P(A \cap B) = 0$

☐ $P(A \cup B) = 1$

☒ Both ☐ and ☐ are true.

☐ None of the above are true.

48. For each of the following scenarios, determine whether the random variable is discrete or continuous.

(a) Dr. Medina records the number of red pens it takes him to grade his students' exams.

☒ Discrete

☐ Continuous

(b) Mr. Lange records the time it takes him to play one round of golf.

☐ Discrete

☒ Continuous

(c) Dr. Medina records the number of Expo marks he uses while teaching over one academic year.

☒ Discrete

☐ Continuous

(d) Mr. Lange records the time it takes him to delete 1000 spam emails.

① Discrete

② Continuous

(e) Dr. Medina records the number of sections of AMAT 108 being taught in any given academic term.

① Discrete

② Continuous

49. Suppose X has a uniform distribution on the interval $(4, 12)$.

(a) Find the height of the density curve. *Hint:* Find the density function of X .

$$(47) \quad f(x) = \frac{1}{12 - 4} = \frac{1}{8} \quad \longrightarrow \quad \text{height} = \frac{1}{8}$$

(b) Compute the probability that X is between 6 and 10 inclusive.

$$(48) \quad P(6 \leq X \leq 10) = \frac{10 - 6}{8}$$

$$(49) \quad = 50\%$$

(c) Compute the probability that X is not less than 9.

$$(50) \quad P(X \geq 9) = \frac{12 - 9}{8}$$

$$(51) \quad = 37.5\%$$

50. Suppose X is a discrete random variable with probability distribution below:

X	4	5	6	7
$p(X)$	0.2	0.1	0.4	0.3

(a) Compute the mean of X .

$$(52) \quad E(X) = 4(0.2) + 5(0.1) + 6(0.4) + 7(0.3)$$

$$(53) \quad = 5.8$$

(b) Compute the standard deviation of X . Round your answer to *three* decimal places.

$$(54) \quad \sigma_X = \sqrt{.2(4 - 5.8)^2 + .1(5 - 5.8)^2 + .4(6 - 5.8)^2 + .3(7 - 5.8)^2}$$

$$(55) \quad = \sqrt{1.16}$$

$$(56) \quad \approx 1.077$$

51. Use the Standard Normal Table, *provided at the back of this packet*, to find all of the following.

(a) $P(76 \leq X \leq 83)$ if X has a normal distribution with mean 80 and standard deviation 2.

$$(57) \quad P(76 \leq X \leq 83) = P\left(\frac{76 - 80}{2} \leq \frac{X - 80}{2} \leq \frac{83 - 80}{2}\right)$$

$$(58) \quad = P(-2 \leq Z \leq 1.5)$$

$$(59) \quad = P(Z \leq 1.5) - P(Z \leq -2)$$

$$(60) \quad \approx 0.9332 - 0.0228$$

$$(61) \quad = 0.9104$$

(b) $P(22 \leq X \leq 24)$ if X has a normal distribution with mean 23 and standard deviation 1.

$$(62) \quad P(22 \leq X \leq 24) = P(22 - 23 \leq X - 23 \leq 24 - 23)$$

$$(63) \quad = P(-1 \leq Z \leq 1)$$

$$(64) \quad = 1 - 2P(Z \leq -1)$$

$$(65) \quad \approx 1 - 2(0.1587)$$

$$(66) \quad = 0.6826$$

(c) $P(-1.89 \leq Z \leq 1.89)$ if Z has a standard normal distribution.

$$(67) \quad P(-1.89 \leq Z \leq 1.89) = 1 - 2P(Z \leq -1.89)$$

$$(68) \quad \approx 1 - 2(0.0294)$$

$$(69) \quad = 0.9412$$

(d) $P(-4 \leq Z \leq 1.48)$ if Z has a standard normal distribution.

$$(70) \quad P(-4 \leq Z \leq 1.48) = P(Z \leq 1.48) - P(Z \leq -4)$$

$$(71) \quad \approx 0.9306 - 0$$

$$(72) \quad = 0.9306$$

52. Suppose Z has a standard normal distribution. Which of the following is equal to $P(-0.96 \leq Z \leq 1.72)$?

① 0.7862

② 0.7897

③ 0.7888

④ 0.7871

53. Suppose X has a uniform distribution on the interval $(3, 12)$. Which of the following is the height of the density curve?

① $1/9$

② $1/10$

③ $1/8$

④ $1/7$

54. Suppose Z has a standard normal distribution. Which of the following is equal to $P(-1.08 \leq Z \leq 0.37)$?

① 0.5057

② 0.5020

③ 0.5052

④ None of the above.

55. Suppose Z has a standard normal distribution. Which of the following is equal to $P(-2.68 \leq Z \leq 1.34)$?

① 0.9061

② 0.9078

③ 0.9077

④ 0.9062

For Questions 56-59, determine whether the statement given is true or false.

56. For every discrete random variable, $P(a \leq x \leq b) = P(a < x < b)$.

① True

② False

57. For a continuous random variable,

$$\mu_X = x_1p(x_1) + x_2p(x_2) + \cdots + x_np(x_n).$$

① True

② False

58. For a continuous random variable,

$$\sigma_X = (x_1 - \mu_X)p(x_1) + (x_2 - \mu_X)p(x_2) + \cdots + (x_n - \mu_X)p(x_n).$$

① True

② False

59. If Z has a standard normal distribution, then Z has mean 1 and standard deviation 0.

① True

② False

60. In a particular major offered at UAlbany, students either have a final exam or a final paper at the end of the semester. The table below gives the probability distribution of the number of final exams the students in the given major will take.

x	0	1	2	3
$p(x)$	0.2	0.15	0.55	0.1

(a) Compute the mean of x .

(73) $\mu = .2(0) + .15(1) + .55(2) + .1(3)$

(74) $= 1.55$

(b) Compute the standard deviation of x .

$$(75) \quad \sigma = \sqrt{.2(0 - 1.55)^2 + .15(1 - 1.55)^2 + .55(2 - 1.55)^2 + .1(3 - 1.55)^2}$$

$$(76) \quad = \sqrt{0.8475}$$

$$(77) \quad \approx 0.921$$

61. Dr. Medina is studying weekly sales of a given type of tablet. He constructs the distribution of sales, which is given below.

X	0	1	2	3
$p(X)$	0.10	0.3	0.45	0.15

(a) Compute the mean of X .

$$(78) \quad \mu = .1(0) + .3(1) + .45(2) + .15(3)$$

$$(79) \quad = 1.65$$

(b) Compute the standard deviation of X .

$$(80) \quad \sigma = \sqrt{.1(0 - 1.65)^2 + .3(1 - 1.65)^2 + .45(2 - 1.65)^2 + .15(3 - 1.65)^2}$$

$$(81) \quad = \sqrt{0.7275}$$

$$(82) \quad \approx 0.853$$

62. Suppose a random sample of size 196 is drawn from a population that has a normal distribution with mean 81.394 and standard deviation 4.395.

(a) Compute the probability that the sample mean is between 80.95 and 81.62.

$$(83) \quad P(80.95 \leq \bar{X} \leq 81.62) = P\left(\frac{80.95 - 81.394}{\frac{4.395}{\sqrt{196}}} \leq \frac{\bar{X} - 81.394}{\frac{4.395}{\sqrt{196}}} \leq \frac{81.62 - 81.394}{\frac{4.395}{\sqrt{196}}}\right)$$

$$(84) \quad \approx P(-1.41 \leq Z \leq 0.72)$$

$$(85) \quad = P(Z \leq 0.72) - P(Z \leq -1.41)$$

$$(86) \quad \approx 0.7642 - 0.0793$$

$$(87) \quad = 0.6849$$

- (b) Compute the probability that the sample mean is between 80.648 and 82.14. *Hint:* This is an example of symmetric limits.

$$\begin{aligned}
 (88) \quad P(80.648 \leq \bar{X} \leq 82.14) &= P\left(\frac{80.648 - 81.394}{\frac{4.395}{\sqrt{196}}} \leq \frac{\bar{X} - 81.394}{\frac{4.395}{\sqrt{196}}} \leq \frac{82.14 - 81.394}{\frac{4.395}{\sqrt{196}}}\right) \\
 (89) \quad &\approx P(-2.38 \leq Z \leq 2.38) \\
 (90) \quad &= 1 - 2P(Z \leq -2.38) \\
 (91) \quad &\approx 1 - 2(.0087) \\
 (92) \quad &= 0.9826
 \end{aligned}$$

Note: The approach in (a) on the previous page may be used. However, we use the symmetry of the limits with respect to zero to simplify our computations.

- (c) Does the sample mean have a normal distribution?

- ① Yes, because the population of interest has a normal distribution.
- ② No, but the sample mean has an approximate normal distribution because the sample size is sufficient.
- ③ No, and we cannot conclude anything about the distribution of the sample mean.

63. Suppose a random sample of size 144 is drawn from a population with $p = 0.47$.

- (a) Compute the probability that the sample proportion is at most 0.45.

$$\begin{aligned}
 (93) \quad P(\hat{p} \leq 0.45) &= P\left(\frac{\hat{p} - 0.47}{\sqrt{\frac{0.47(1 - 0.47)}{144}}} \leq \frac{0.45 - 0.47}{\sqrt{\frac{0.47(1 - 0.47)}{144}}}\right) \\
 (94) \quad &\approx P(Z \leq -0.48) \\
 (95) \quad &\approx 0.3156
 \end{aligned}$$

- (b) Compute the probability that the sample proportion is between 0.375 and 0.565. *Hint:* This is an example of symmetric limits.

$$\begin{aligned}
 (96) \quad P(0.375 \leq \hat{p} \leq 0.565) &= P\left(\frac{0.375 - 0.47}{\sqrt{\frac{0.47(1 - 0.47)}{144}}} \leq \frac{\hat{p} - 0.47}{\sqrt{\frac{0.47(1 - 0.47)}{144}}} \leq \frac{0.565 - 0.47}{\sqrt{\frac{0.47(1 - 0.47)}{144}}}\right) \\
 (97) \quad &\approx P(-2.28 \leq Z \leq 2.28) \\
 (98) \quad &= 1 - 2P(Z \leq -2.28) \\
 (99) \quad &\approx 1 - 2(0.0113) \\
 (100) \quad &= 0.9774
 \end{aligned}$$

(c) Does the sample proportion have a normal distribution?

- ① Yes, because the population of interest has a normal distribution.
- ② No, but the sample proportion has an approximate normal distribution because the sample size is sufficient.
- ③ No, but the sample proportion has an approximate normal distribution because the success-failure conditions are met.
- ④ No, and we cannot conclude anything about the distribution of the sample proportion.

64. Dr. Medina is studying student performance in this course across all sections being taught. To do this, he gathers 3136 prior scores from exams covering probability and distributions. Assume that the population of interest is normally distributed with mean 75 and standard deviation 2.614.

(a) Compute the probability that the sample mean is between 74.93 and 75.05.

$$\begin{aligned}(101) \quad P(74.93 \leq \bar{X} \leq 75.05) &= P\left(\frac{74.93 - 75}{\frac{2.614}{\sqrt{3136}}} \leq \frac{\bar{X} - 75}{\frac{2.614}{\sqrt{3136}}} \leq \frac{75.05 - 75}{\frac{2.614}{\sqrt{3136}}}\right) \\(102) \quad &\approx P(-1.5 \leq Z \leq 1.07) \\(103) \quad &= P(Z \leq 1.07) - P(Z \leq -1.5) \\(104) \quad &\approx 0.8577 - 0.0668 \\(105) \quad &= 0.7909\end{aligned}$$

(b) Compute the probability that the sample mean is between 74.88 and 75.12. *Hint:* This is an example of symmetric limits.

$$\begin{aligned}(106) \quad P(74.88 \leq \bar{X} \leq 75.12) &= P\left(\frac{74.88 - 75}{\frac{2.614}{\sqrt{3136}}} \leq \frac{\bar{X} - 75}{\frac{2.614}{\sqrt{3136}}} \leq \frac{75.12 - 75}{\frac{2.614}{\sqrt{3136}}}\right) \\(107) \quad &\approx P(-2.57 \leq Z \leq 2.57) \\(108) \quad &= 1 - 2P(Z \leq -2.57) \\(109) \quad &\approx 1 - 2(0.0051) \\(110) \quad &= 0.9898\end{aligned}$$

(c) Does the sample mean have a normal distribution?

- ① Yes, because the population of interest has a normal distribution.
- ② No, but the sample mean has an approximate normal distribution because the sample size is sufficient.
- ③ No, and we cannot conclude anything about the distribution of the sample mean.

65. In professional men's tennis, it is a widely-held belief that 78% of players who have a first-serve percentage (the percentage of his first serves that land in the service box) of over 60% in any set go on to win the set. Suppose a random sample of 64 professional men's tennis players is collected.

- (a) Compute the probability that the percentage of the sample who have a first-serve percentage over 60% and went on to win the set is between 75% and 79%.

$$(111) \quad P(0.75 \leq \hat{p} \leq 0.79) = P\left(\frac{0.75 - 0.78}{\sqrt{\frac{0.78(1-0.78)}{64}}} \leq \frac{\hat{p} - 0.78}{\sqrt{\frac{0.78(1-0.78)}{64}}} \leq \frac{0.79 - 0.78}{\sqrt{\frac{0.78(1-0.78)}{64}}}\right)$$

$$(112) \quad \approx P(-0.58 \leq Z \leq 0.19)$$

$$(113) \quad = P(Z \leq 0.19) - P(Z \leq -0.58)$$

$$(114) \quad \approx 0.5753 - 0.2810$$

$$(115) \quad = 0.2943$$

- (b) Compute the probability that the percentage of the sample who have a first-serve percentage over 60% and went on to win the set is between 68% and 88%. *Hint:* This is an example of symmetric limits.

$$(116) \quad P(0.68 \leq \hat{p} \leq 0.88) = P\left(\frac{0.68 - 0.78}{\sqrt{\frac{0.78(1-0.78)}{64}}} \leq \frac{\hat{p} - 0.78}{\sqrt{\frac{0.78(1-0.78)}{64}}} \leq \frac{0.88 - 0.78}{\sqrt{\frac{0.78(1-0.78)}{64}}}\right)$$

$$(117) \quad \approx P(-1.93 \leq Z \leq 1.93)$$

$$(118) \quad = 1 - 2P(Z \leq -1.93)$$

$$(119) \quad \approx 1 - 2(0.0268)$$

$$(120) \quad = 0.9464$$

- (c) Does the sample proportion have a normal distribution?

① Yes, because the population of interest has a normal distribution.

② No, but the sample proportion has an approximate normal distribution because the sample size is sufficient.

③ No, but the sample proportion has an approximate normal distribution because the success-failure conditions are met.

④ No, and we cannot conclude anything about the distribution of the sample proportion.

66. Suppose a random sample of size n is chosen from a population of interest that has a normal distribution with mean μ and standard deviation σ . Which of the following is true?

- ① $\bar{X}$ has a normal distribution, irrespective of n . ② The Central Limit Theorem gives $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ has a standard normal distribution.
- ③ Both ① and ② are true. ④ None of the above are true.

67. Suppose the success-failure conditions $np \geq 10$ and $n(1 - p) \geq 10$ are true for a given sample. Which of the following is true about the distribution of $\hat{p}$?

- ① It is uniform. ② It is approximately normal.
- ③ It is normal. ④ None of the above are true.

UNIT 3: Chapter 9 of Peck, Short, and Olsen

68. Use the Central Area Captured Table or the T Probabilities Table, *both provided at the back of this packet*, to find all of the following.

- (a) $P(-3.61 \leq X \leq 3.61)$ if X has a t distribution with 18 degrees of freedom **0.998**
- (b) $P(-2.39 \leq X \leq 2.39)$ if X has a t distribution with 60 degrees of freedom **0.98**
- (c) $P(X < -2.90)$ if X has a t distribution with 8 degrees of freedom **0.009 b/c symmetry of T**
- (d) $P(X > 2.9)$ if X has a t distribution with 23 degrees of freedom **0.004**
- (e) $P(X > 3.4)$ if X has a t distribution with 6 degrees of freedom **0.007**
- (f) $P(X < -2.6)$ if X has a t distribution with 10 degrees of freedom **0.013 b/c symmetry of T**
- (g) $P(-2.3 \leq X \leq 2.3)$ if X has a t distribution with 24 degrees of freedom **$1 - 2(0.015) = 0.97$ b/c symmetry of T**
- (h) $P(-2.9 \leq X \leq 2.9)$ if X has a t distribution with 26 degrees of freedom **$1 - 2(0.004) = 0.992$ b/c symmetry of T**

69. A random sample of eight observations is given below:

73 77 75 74 76 78 71 73

Find a reasonable point estimate for μ . Round *both* answers to *three* decimal places, where appropriate. A reasonable point estimate for μ is $\bar{x}$. We can also use the sample median, but only when the data is symmetric.

$$(121) \quad \bar{x} = \frac{73 + 77 + 75 + 74 + 76 + 78 + 71 + 73}{8}$$

$$(122) \quad = 74.625$$

70. Dr. Medina is interested in studying the frequency of spam emails that the Math Department's faculty receives. A random sample of 64328 emails from multiple Department faculty members indicates that 18 out of every 48 emails in his inbox are spam.

(a) Find a reasonable point estimate for p . A reasonable point estimate for the population proportion is the sample proportion. As a result,

$$(123) \quad \hat{p} = \frac{18}{48} = \frac{3}{8}.$$

(b) Dr. Medina wishes to find a central 97% confidence interval for p . Are all requirements for the Large-Sample Confidence Interval for p met?

① Yes, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.

② Yes, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

③ No, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.

④ No, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

(c) Find the 97% z^* critical number. *Hint:* Use the Central Area Captured table. The 97% z^* critical number is 2.170.

(d) Construct a central 97% confidence interval for p . Round *both* endpoints to *three* decimal places.

$$(124) \quad \frac{3}{8} \pm 2.170 \sqrt{\frac{\frac{3}{8} \left(1 - \frac{3}{8}\right)}{64328}} = \left(\frac{3}{8} - 2.170 \sqrt{\frac{\frac{3}{8} \left(1 - \frac{3}{8}\right)}{64328}}, \frac{3}{8} + 2.170 \sqrt{\frac{\frac{3}{8} \left(1 - \frac{3}{8}\right)}{64328}} \right)$$

$$(125) \quad \approx (0.371, 0.379)$$

-
- (e) Find the minimum sample size required to conservatively estimate p within .005 with 97% confidence.

$$(126) \quad n = \frac{1}{4} \left(\frac{2.170}{.005} \right)^2$$

$$(127) \quad = 47089$$

Note: The sample size is not rounded up because an integer result was obtained.

71. Mr. Lange is interested in studying scoring at NBA games. To do this, he selects a random sample of 15 games and observes the number of points each team scores in regulation. The sample is found to have mean 105.739 and standard deviation 4.934, correct to three decimal places.

- (a) Mr. Lange wishes to construct a central 96% confidence interval for μ , the population mean. Are all requirements met?

① Yes, because the population of interest has a normal distribution.

② Yes, because the selected sample has a normal distribution.

③ Yes, because the sample size is sufficient.

④ No, because the sample size is insufficient and we do not know that the population or the sample has a normal distribution.

Note: Because there are two teams playing in each of 15 games, the total sample size is 30 and, thus, is sufficiently large.

- (b) Find an appropriate critical number associated with a central 96% confidence interval. *Hint:* Use the Central Area Captured table. The 96% t^* critical number with 29 degrees of freedom is 2.150.

- (c) Construct a central 96% confidence interval for μ . Round both endpoints to three decimal places.

$$(128) \quad 105.739 \pm \frac{2.150(4.934)}{\sqrt{15}} = \left(105.739 - \frac{2.150(4.934)}{\sqrt{15}}, 105.739 + \frac{2.150(4.934)}{\sqrt{15}} \right)$$

$$(129) \quad \approx (103, 108.478)$$

72. Suppose X has a t distribution with 20 degrees of freedom. Which of the following is equal to $P(-3 \leq X \leq 3)$?

① 0.950

② 0.980

③ 0.992

④ 0.960

73. Suppose X has a t distribution with 13 degrees of freedom. Which of the following is equal to $P(-2.3 \leq X \leq 2.3)$?

① 0.962

② 0.990

③ 0.999

④ None of the above.

74. A random sample of 101 observations is drawn from a population that has a normal distribution. The sample is found to have mean 76.923 and standard deviation 1.321. Mr. Lange wishes to find a central 98% confidence interval for μ , the population mean.

(a) Are all requirements for the One-Sample Confidence Interval for μ met?

① Yes, because the population of interest has a normal distribution.

② Yes, because the selected sample has a normal distribution.

③ Yes, because the sample size is sufficient.

④ No, because the sample size is insufficient and we do not know that the population or the sample has a normal distribution.

(b) Find an appropriate critical number associated with a central 98% confidence interval. *Hint:* Use the Central Area Captured Table. **An appropriate 98% t^* critical number is 2.364.**

(c) Construct a central 98% confidence interval for μ . Round *both* endpoints to *three* decimal places.

$$(130) \quad 76.923 \pm \frac{2.364(1.321)}{\sqrt{101}} = \left(76.923 - \frac{2.364(1.321)}{\sqrt{101}}, 76.923 + \frac{2.364(1.321)}{\sqrt{101}} \right)$$

$$(131) \quad \approx (76.612, 77.234)$$

75. Dr. Medina is studying the amount of time it takes to walk a mile. A random sample of 71 miles walked shows that 43% of them take less than 15 minutes.

(a) Dr. Medina wishes to find a central 99% confidence interval for p . Are all requirements of the Large-Sample Confidence Interval for p met?

① Yes, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.

② Yes, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

③ No, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.

④ No, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

(b) Find the 99% z^* critical number. *Hint:* Use the Central Area Captured Table. The 99% z^* critical number is 2.576.

(c) Construct a central 99% confidence interval for p . Round *both* endpoints to *three* decimal places.

(132)

$$0.43 \pm 2.576 \sqrt{\frac{0.43(1 - 0.43)}{71}} = \left(0.43 - 2.576 \sqrt{\frac{0.43(1 - 0.43)}{71}}, 0.43 + 2.576 \sqrt{\frac{0.43(1 - 0.43)}{71}} \right)$$

(133)

$$\approx (0.279, 0.581)$$

(d) Find the minimum sample size required to conservatively estimate p within 0.003 with 99% confidence.

(134)

$$n = \frac{1}{4} \left(\frac{2.576}{0.003} \right)^2$$

(135)

$$\approx 184327.111$$

(136)

$$\uparrow 184328$$

76. Mr. Lange surveys 639 professional athletes on the amount of time they spend training. The survey finds that 62% train for more than four hours each day. A reasonable point estimate for the population proportion is...

① ... all college instructors.

② ... the 438 college instructors.

③ ... 62%

④ ... 38%

UNIT 4: Chapters 10 and 11 of Peck, Short, and Olsen

77. Of the claims shown below, determine which are valid statistical hypotheses and which are not.

(a) $p = 0.5$

① Valid

② Invalid

(b) $\bar{x} > 47$

① Valid

② Invalid

(c) $\mu < 82$

① Valid

② Invalid

(d) $\hat{p} = 0.423$

① Valid

② Invalid

(e) $\mu \neq 78$

① Valid

② Invalid

(f) $\bar{x} < 83$

① Valid

② Invalid

78. Consider the scenario from Problem 70. Prior information suggests the proportion of spam emails that the Math Department's faculty receives is 38%. Dr. Medina wishes to know if there is enough evidence to conclude that the prior information is an overestimate.

(a) Select the correct pair of statistical hypotheses.

① $H_0: p = 0.38$
vs.
 $H_1: p < 0.38$

② $H_0: p = 0.38$
vs.
 $H_1: p > 0.38$

③ $H_0: p = 0.38$
vs.
 $H_1: p \neq 0.38$

(b) Are the requirements for the One-Sample Z Test for p met?

① Yes, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.

② Yes, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

③ No, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.

④ No, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

(c) Compute the z^* test statistic. Round your answer to *two* decimal places.

$$(137) \quad z^* = \frac{\frac{18}{48} - 0.38}{\sqrt{\frac{0.38(1 - 0.38)}{64328}}}$$

$$(138) \quad \approx -2.61^{**}$$

(d) Which of the following correctly describes the distribution of the test statistic?

① It has a normal distribution with mean 1 and standard deviation 2.

② It has a standard normal distribution.

③ It has a normal distribution with mean 1 and standard deviation 0.

④ It has a normal distribution with mean 2 and standard deviation 1.

(e) Compute the p -value, as discussed in class, and express your answer to *four* decimal places.

$$(139) \quad \alpha_0 = \text{P-value} = P_{H_0}(Z \leq -2.61)$$

$$(140) \quad \approx 0.0045$$

Note: Some advanced statistics textbooks might show (140) as $\alpha_0 = P_{H_0}(N(0, 1) \leq -2.61)$.

(f) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the prior information is an overestimate.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the prior information is an overestimate.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the prior information is an overestimate.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the prior information is an overestimate.

79. In an effort to learn about TV malfunction rates, Mr. Lange selects 121 of a particular brand and records the number of malfunctions they have in the first 10 years. While the manufacturer claims that the TVs should only malfunction once in the first 10 years, the sampled TVs malfunction on average 2.968 times in the first 10 years, with a standard deviation of 1.024 times. Mr. Lange wishes to know if there is enough evidence to conclude that the prior information is an underestimate.

(a) Select the correct pair of statistical hypotheses.

$$\textcircled{1} \quad \begin{array}{c} H_0: \mu = 1 \\ \text{vs.} \\ H_1: \mu < 1 \end{array}$$

$$\textcircled{2} \quad \begin{array}{c} H_0: \mu = 1 \\ \text{vs.} \\ H_1: \mu > 1 \end{array}$$

$$\textcircled{3} \quad \begin{array}{c} H_0: \mu = 1 \\ \text{vs.} \\ H_1: \mu \neq 1 \end{array}$$

(b) Are the requirements for the One-Sample T Test for μ met?

- ① Yes, because the population of interest has a normal distribution.
- ② Yes, because the selected sample has a normal distribution.
- ③ Yes, because the sample size is sufficient.
- ④ No, because the sample size is insufficient and we do not know that the population or the sample has a normal distribution.

(c) Compute the t^* test statistic. Round your answer to *one* decimal place.

$$(141) \quad t^* = \frac{2.968 - 1}{\frac{1.024}{\sqrt{121}}}$$

$$(142) \quad \approx 21.1^{***}$$

(d) The test statistic has a t distribution with how many degrees of freedom?

① 121

② 122

③ 119

④ 120

(e) Compute the p -value, as discussed in class, and express your answer to *three* decimal places.

$$(143) \quad \alpha_0 = \text{P-value} = P_{H_0}(T \geq 21.1)$$

$$(144) \quad \approx 0.$$

Note: Some advanced statistics textbooks might show (144) as $\alpha_0 = P_{H_0}(t(120) \geq 21.1)$.

(f) Which of the following is the correct conclusion?

① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the claim is an underestimate.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the claim is an underestimate.

③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the claim is an underestimate.

④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the claim is an underestimate.

80. Consider the scenario from the previous problem. Suppose the manufacturer claims that 73% of TVs will not have a malfunction in the first 10 years. However, the selected sample shows that only 65% of TVs did not malfunction in the first 10 years. Mr. Lange wishes to determine if there is enough evidence to conclude that the claim is incorrect.

(a) Select the correct pair of statistical hypotheses.

① $H_0: p = 0.73$
vs.
 $H_1: p < 0.73$

② $H_0: p = 0.73$
vs.
 $H_1: p > 0.73$

③ $H_0: p = 0.73$
vs.
 $H_1: p \neq 0.73$

(b) Are all the requirements of the One-Sample Z Test for p met?

- ① Yes, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.
- ② Yes, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.
- ③ No, because $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$.
- ④ No, because one of $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ is false.

(c) Compute the z^* test statistic, as discussed in class. Round your answer to *two* decimal places, where appropriate.

$$\begin{aligned} (145) \quad z^* &= \frac{0.65 - 0.73}{\sqrt{\frac{0.73(1 - 0.73)}{121}}} \\ (146) \quad &\approx -1.98^* \end{aligned}$$

(d) Which of the following correctly describes the distribution of the test statistic?

- ① It has a normal distribution with mean 1 and standard deviation 2.
- ② It has a normal distribution with mean 1 and standard deviation 0.
- ③ It has a standard normal distribution.
- ④ It has a normal distribution with mean 2 and standard deviation 1.

(e) Compute the p -value as discussed in class, and express your answer to *four* decimal places.

$$\begin{aligned} (147) \quad \alpha_0 = \text{P-value} &= 2P_{H_0}(Z \leq -1.98) \\ (148) \quad &\approx 2(0.0239) \\ (149) \quad &= 0.0478 \end{aligned}$$

Note: Some advanced statistics textbooks might show (149) as $\alpha_0 = 2P_{H_0}(N(0, 1) \leq -1.98)$.

(f) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the claim is incorrect.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the claim is incorrect.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the claim is incorrect.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the claim is incorrect.

81. A famous drug manufacturer claims that its new cholesterol medication, along with diet and exercise, gets patients' total cholesterol to measure at most 160. To verify or refute this claim, Dr. Medina conducts a clinical trial with 537 participants. Each is given the new medication at a dosage prescribed by their cardiologist, and cholesterol levels are tested in each participant after eight months. The participants' eight-month total cholesterol levels average 161.483 with a standard deviation of 16.938. Dr. Medina wishes to use the One-Sample T Test for μ to determine whether there is enough evidence to conclude that the claim is an underestimate.

(a) Select the correct pair of statistical hypotheses.

$$\textcircled{1} \quad \begin{array}{c} H_0: \mu = 160 \\ \text{vs.} \\ H_1: \mu < 160 \end{array}$$

$$\textcircled{2} \quad \begin{array}{c} H_0: \mu = 160 \\ \text{vs.} \\ H_1: \mu > 160 \end{array}$$

$$\textcircled{3} \quad \begin{array}{c} H_0: \mu = 160 \\ \text{vs.} \\ H_1: \mu \neq 160 \end{array}$$

(b) Are all the requirements for the One-Sample T Test for μ met?

$\textcircled{1}$ Yes, because the population of interest has a normal distribution.

$\textcircled{2}$ Yes, because the selected sample has a normal distribution.

$\textcircled{3}$ Yes, because the sample size is sufficient.

$\textcircled{4}$ No, because the sample size is insufficient and we do not know that the population or the sample has a normal distribution.

(c) Compute the t^* test statistic. Round your answer to *one* decimal place.

$$(150) \quad t^* = \frac{161.483 - 160}{\frac{16.938}{\sqrt{537}}}$$

$$(151) \quad \approx 2.0^*$$

(d) The test statistic has a t distribution with how many degrees of freedom?

$\textcircled{1}$ 538

$\textcircled{2}$ 537

$\textcircled{3}$ 536

$\textcircled{4}$ 535

(e) Compute the p -value as discussed in class, and express your answer to *three* decimal places.

$$(152) \quad \alpha_0 = \text{P-value} = P_{H_0}(T \geq 2.0)$$

$$(153) \quad \approx P_{H_0}(Z \geq 2.0)$$

$$(154) \quad \approx 0.023$$

Note:

- Some advanced statistics textbooks might show (154) as $\alpha_0 = P_{H_0}(t(536) \geq 2.0)$.
- The approximation $t(536) \approx N(0, 1)$ is made because $df = 536 > 120$.

(f) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the claim is an underestimate.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the claim is an underestimate.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the claim is an underestimate.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the claim is an underestimate.

82. After experiencing his fourth light bulb failure in six months, Mr. Lange is seeking answers. The manufacturer of the brand of light bulb he uses for his desk lamp claims that their light bulbs fail at the same rate as their competitors' bulbs. Wanting answers, Mr. Lange tests 24912 bulbs from the manufacturer and 24912 bulbs from the competitors. In six months' time, 8112 competitors' bulbs fail, but 9076 manufacturer bulbs fail in the same time frame. Mr. Lange wishes to use the 2-Sample Z Test for Equal Proportions to determine whether there is enough evidence to conclude that the manufacturer's claim is an underestimate. *Use p_1 for the proportion of manufacturer bulbs that fail in six months and p_2 for the proportion of competitor bulbs that fail in six months.*

(a) Select the correct pair of statistical hypotheses.

- | | | | | | |
|---|---|---|---|---|--|
| ① | $H_0: p_1 = p_2$
vs.
$H_1: p_1 < p_2$ | ② | $H_0: p_1 = p_2$
vs.
$H_1: p_1 > p_2$ | ③ | $H_0: p_1 = p_2$
vs.
$H_1: p_1 \neq p_2$ |
|---|---|---|---|---|--|

(b) Are the requirements for the 2-Sample Z Test for Equal Proportions met?

- ① Yes, because both populations of interest have normal distributions.
- ① Yes, because both sample sizes are sufficient.
- ③ Yes, because both sets of success-failure conditions are met.
- ④ No, because none of the above apply.

(c) Compute the pooled sample proportion and round your answer to *five* decimal places.

(155)
$$\hat{p}_c = \frac{9076 + 8112}{24912 + 24912}$$

(156)
$$\approx 0.34497$$

(d) Compute the z^* test statistic. Round your answer to *two* decimal places.

$$\begin{aligned} (157) \quad z^* &= \frac{\frac{9076}{24912} - \frac{8112}{24912}}{\sqrt{\frac{0.34497(1 - 0.34497)}{24912} + \frac{0.34497(1 - 0.34497)}{24912}}} \\ (158) \quad &\approx 9.09^{***} \end{aligned}$$

(e) Which of the following correctly describes the distribution of the test statistic?

- | | |
|--|--|
| ① It has a normal distribution with mean and standard deviation both equal to 1. | ② It has a standard normal distribution. |
| ③ It has a normal distribution with mean and standard deviation both equal to 3. | ④ It has a normal distribution with mean and standard deviation both equal to 2. |

(f) Compute the p -value as discussed in class, and express your answer to *four* decimal places.

$$\begin{aligned} (159) \quad \alpha_0 = \text{P-value} &= P_{H_0}(Z \geq 9.09) \\ (160) \quad &\approx 0. \end{aligned}$$

(g) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the claim is an underestimate.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the claim is an underestimate.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the claim is an underestimate.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the claim is an underestimate.

83. Mr. Lange is an avid golfer. For years, he has suspected that he plays better (scores lower) on Sundays than on Saturdays. To prove or disprove this notion, he selects a number of scores from both days, all between 2008 and 2024. Summary statistics are shown below.

Day	Average	Variance	Count
Saturday	77.635	20.162	203
Sunday	76.528	19.981	248

For this problem, assume all scores selected for analysis are randomly selected from a normal population. Mr. Lange wishes to know if there is enough evidence to conclude that he plays better on Sundays than on Saturdays. Use μ_1 to represent his overall average on Sundays and μ_2 to represent his overall average on Saturdays.

- (a) Select the correct pair of statistical hypotheses.

1 $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 < \mu_2$	2 $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 > \mu_2$	3 $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$
--	--	--

- (b) Are the requirements for Welch's T Test met?

1 Yes	2 No
---	--

- (c) Compute df for this test as discussed in class.

(161)	$df = \min\{203, 248\} - 1$
(162)	$= 203 - 1$
(163)	$= 202$

- (d) Compute the t^* test statistic. Round your answer to *one* decimal place.

(164)	$t^* = \frac{76.528 - 77.635}{\sqrt{\frac{19.981}{248} + \frac{20.162}{203}}}$
(165)	$\approx -2.6^{**}$

- (e) Compute the p -value as discussed in class, and express your answer to *three* decimal places.

(166)	$\alpha_0 = \text{P-value} = P_{H_0}(T \leq -2.6)$
(167)	$\approx P_{H_0}(Z \geq 2.6)$
(168)	≈ 0.005

Note: The jump from (167) to (168) is made by symmetry and the approximation $t(202) \approx N(0, 1)$ because $df > 120$.

(f) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that Mr. Lange plays better on Sundays than on Saturdays.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that Mr. Lange plays better on Sundays than on Saturdays.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that Mr. Lange plays better on Sundays than on Saturdays.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that Mr. Lange plays better on Sundays than on Saturdays.

84. A car dealership is noticing that one particular make and model year of car, Model A, is consistently in their repair shop for a turned-on check engine light. In theory, the frequency of make and model year of a car having a turned-on check engine light should be consistent. To test this theory, 527 Model A cars and 492 other cars that are in the repair shop are selected at random. It is found that 230 Model A cars and 198 other cars have a turned-on check engine light. The dealership wishes to know if there is enough evidence to conclude that the theory is incorrect. Use p_1 for the proportion of Model A cars that have a turned-on check engine light and p_2 for the proportion of other cars that have a turned-on check engine light.

(a) Select the correct pair of statistical hypotheses.

- | | | | | | |
|---|---|---|---|---|--|
| ① | $H_0: p_1 = p_2$
vs.
$H_1: p_1 < p_2$ | ② | $H_0: p_1 = p_2$
vs.
$H_1: p_1 > p_2$ | ③ | $H_0: p_1 = p_2$
vs.
$H_1: p_1 \neq p_2$ |
|---|---|---|---|---|--|

(b) Are all requirements of the Two-Sample Z Test for Equal Proportions met?

- ① Yes, because both populations of interest have normal distributions.
- ① Yes, because both sample sizes are sufficient.
- ③ Yes, because both sets of success-failure conditions are met.
- ④ No, because none of the above apply.

(c) Compute the pooled sample proportion and round your answer to *five* decimal places.

$$\begin{aligned} (169) \quad \hat{p}_c &= \frac{230 + 198}{527 + 492} \\ (170) \quad &\approx 0.42002 \end{aligned}$$

-
- (d) Compute the z^* test statistic, as discussed in class. Round your answer to *two* decimal places, where appropriate.

$$\begin{aligned} (171) \quad z^* &= \frac{\frac{230}{527} - \frac{198}{492}}{\sqrt{\frac{0.42002(1 - 0.42002)}{527} + \frac{0.42002(1 - 0.42002)}{492}}} \\ (172) \quad &\approx 1.10 \end{aligned}$$

- (e) Which of the following correctly describes the distribution of the test statistic?

- ① It has a standard normal distribution. ② It has a normal distribution with mean 1 and standard deviation 0.
- ③ It has a normal distribution with mean 1 and standard deviation 2. ④ It has a normal distribution with mean 2 and standard deviation 1.

- (f) Compute the p -value as discussed in class, and express your answer to *four* decimal places.

$$\begin{aligned} (173) \quad \alpha_0 = \text{P-value} &= 2P_{H_0}(Z \leq -1.10) \\ (174) \quad &\approx 2(0.1357) \\ (175) \quad &= 0.2714 \end{aligned}$$

- (g) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the theory is incorrect.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the theory is incorrect.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the theory is incorrect.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the theory is incorrect.

(f) Which of the following is the correct conclusion?

- ① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that Brand K printers malfunction faster than normal.
- ② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that Brand K printers malfunction faster than normal.
- ③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that Brand K printers malfunction faster than normal.
- ④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that Brand K printers malfunction faster than normal.

UNIT 5: Chapter 5 of Peck, Short, and Olsen

86. Consider the bivariate data on variables X and Y .

$X :$	3	5	4	6	8
$Y :$	4	3	1	5	2

(a) Compute $\bar{x}$ and $\bar{y}$.

$$(181) \quad \bar{x} = \frac{3 + 5 + 4 + 6 + 8}{5} = 5.2 \qquad \bar{y} = \frac{4 + 3 + 1 + 5 + 2}{5} = 3$$

(b) Compute s_x and s_y . Round your answers to *three* decimal places.

$$(182) \quad s_x^2 = \frac{(3 - 5.2)^2 + (5 - 5.2)^2 + (4 - 5.2)^2 + (6 - 5.2)^2 + (8 - 5.2)^2}{4}$$

$$(183) \quad = 3.7 \longrightarrow s_x \approx 1.924$$

$$(184) \quad s_y^2 = \frac{(4 - 3)^2 + (3 - 3)^2 + (1 - 3)^2 + (5 - 3)^2 + (2 - 3)^2}{4}$$

$$(185) \quad = 2.5 \longrightarrow s_y \approx 1.581$$

(c) Compute the sample correlation coefficient. Round your answer to *three* decimal places.

$$(186) \quad r = \frac{(3 - 5.2)(4 - 3) + (5 - 5.2)(3 - 3) + (4 - 5.2)(1 - 3) + (6 - 5.2)(5 - 3) + (8 - 5.2)(2 - 3)}{4(1.924)(1.581)}$$

$$(187) \quad \approx -0.082$$

(d) Find the least-squares regression line for the data. Round *both* coefficients to *three* decimal places.

$$(188) \quad b = \frac{(-0.082)(1.581)}{1.924} \approx -0.067 \qquad a = 3 - (-0.067)(5.2) \approx 3.348$$

$$(189) \quad \hat{Y} = 3.348 - 0.067X$$

87. Suppose a bivariate data set yields $r = 0.989$. Which of the following is *not* true?

- ① X and Y have a strong positive relationship. ② Changes in X cause changes in Y .
- ③ At least one residual is not zero. ④ All of the above are true.

88. Below are five observations on variables X and Y .

$X :$	4	9	5	3	6
$Y :$	1	5	4	2	3

(a) The sample is found to have $\bar{y} = 3$. Compute $\bar{x}$.

$$\begin{aligned} (190) \quad \bar{x} &= \frac{4 + 9 + 5 + 3 + 6}{5} \\ (191) \quad &= 5.4 \end{aligned}$$

(b) The sample is found to have $s_y^2 = 2.5$. Compute s_x^2 .

$$\begin{aligned} (192) \quad s_x^2 &= \frac{(4 - 5.4)^2 + (9 - 5.4)^2 + (5 - 5.4)^2 + (3 - 5.4)^2 + (6 - 5.4)^2}{4} \\ (193) \quad &= 5.3 \end{aligned}$$

(c) The sample is found to have $s_y \approx 1.581$. Compute s_x and round your answer to *three* decimal places.

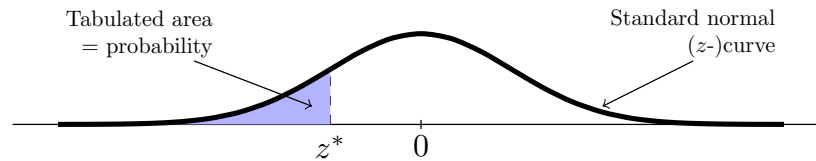
$$\begin{aligned} (194) \quad s &= \sqrt{5.3} \\ (195) \quad &\approx 2.302 \end{aligned}$$

(d) Compute the sample correlation coefficient. Round your answer to *three* decimal places.

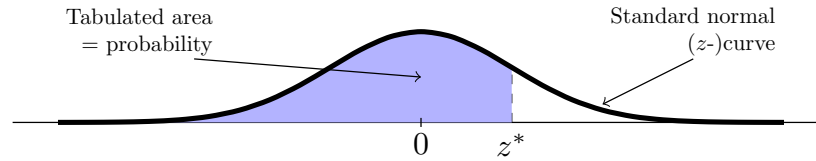
$$\begin{aligned} (196) \quad r &= \frac{(4 - 5.4)(1 - 3) + (9 - 5.4)(5 - 3) + (5 - 5.4)(4 - 3) + (3 - 5.4)(2 - 3) + (6 - 5.4)(3 - 3)}{4(1.581)(2.302)} \\ (197) \quad &\approx 0.824 \end{aligned}$$

(e) Find the least-squares regression line for the data. Round *both* coefficients to *three* decimal places.

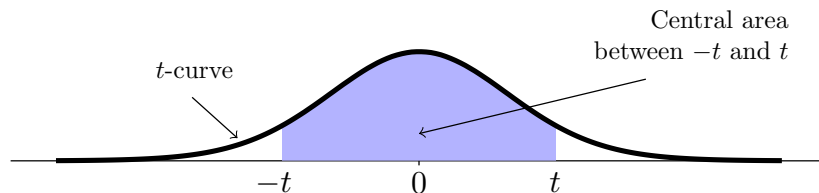
$$\begin{aligned} (198) \quad b &= \frac{0.824(1.581)}{2.302} \approx 0.566 & a &= 3 - 5.4(0.566) \approx -0.056 \\ (199) \quad \hat{Y} &= 0.566X - 0.056 \end{aligned}$$

Standard Normal Table

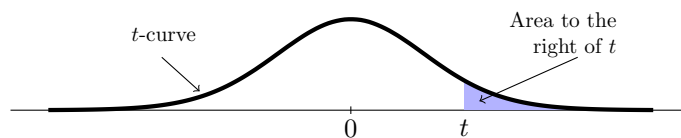
z^*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Table

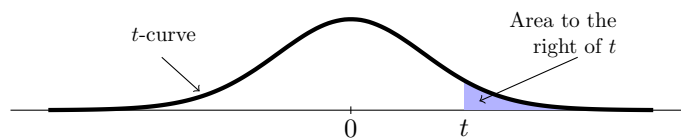
z^*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

t-Distribution Table of Critical Values

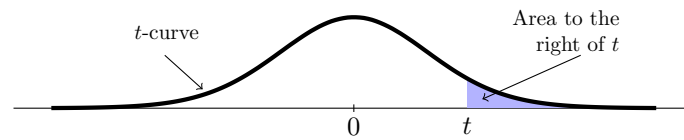
degree of freedom	Central Area Captured / Confidence Level								
	80%	90%	95%	96%	97%	98%	99%	99.8%	99.9%
1	3.078	6.314	12.706	15.895	21.205	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	4.849	5.643	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	3.482	3.896	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	2.999	3.298	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	2.757	3.003	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	2.612	2.829	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.517	2.715	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.449	2.634	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.398	2.574	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.359	2.527	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.328	2.491	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.303	2.461	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.282	2.436	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.264	2.415	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.249	2.397	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.235	2.382	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.224	2.368	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.214	2.356	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.205	2.346	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.197	2.336	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.189	2.328	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.183	2.320	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.177	2.313	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.172	2.307	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.167	2.301	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.162	2.296	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.158	2.291	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.154	2.286	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.150	2.282	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.147	2.278	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.123	2.250	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.109	2.234	2.403	2.678	3.261	3.496
60	1.296	1.671	2.000	2.099	2.223	2.390	2.660	3.232	3.460
70	1.294	1.667	1.994	2.093	2.215	2.381	2.648	3.211	3.435
80	1.292	1.664	1.990	2.088	2.209	2.374	2.639	3.195	3.416
90	1.291	1.662	1.987	2.084	2.205	2.368	2.632	3.183	3.402
100	1.290	1.660	1.984	2.081	2.201	2.364	2.626	3.174	3.390
110	1.289	1.659	1.982	2.078	2.199	2.361	2.621	3.166	3.381
120	1.289	1.658	1.980	2.076	2.196	2.358	2.617	3.160	3.373
<i>z</i>-critical = ∞	1.282	1.645	1.960	2.054	2.170	2.326	2.576	3.090	3.291

Tail Areas for t Curves

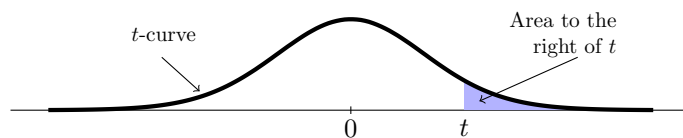
$t^* \backslash df$	1	2	3	4	5	6	7	8	9	10
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.468	0.465	0.463	0.463	0.462	0.462	0.462	0.461	0.461	0.461
0.2	0.437	0.430	0.427	0.426	0.425	0.424	0.424	0.423	0.423	0.423
0.3	0.407	0.396	0.392	0.390	0.388	0.387	0.386	0.386	0.385	0.385
0.4	0.379	0.364	0.358	0.355	0.353	0.352	0.351	0.350	0.349	0.349
0.5	0.352	0.333	0.326	0.322	0.319	0.317	0.316	0.315	0.315	0.314
0.6	0.328	0.305	0.295	0.290	0.287	0.285	0.284	0.283	0.282	0.281
0.7	0.306	0.278	0.267	0.261	0.258	0.255	0.253	0.252	0.251	0.250
0.8	0.285	0.254	0.241	0.234	0.230	0.227	0.225	0.223	0.222	0.221
0.9	0.267	0.232	0.217	0.210	0.205	0.201	0.199	0.197	0.196	0.195
1.0	0.250	0.211	0.196	0.187	0.182	0.178	0.175	0.173	0.172	0.170
1.1	0.235	0.193	0.176	0.167	0.161	0.157	0.154	0.152	0.150	0.149
1.2	0.221	0.177	0.158	0.148	0.142	0.138	0.135	0.132	0.130	0.129
1.3	0.209	0.162	0.142	0.132	0.125	0.121	0.117	0.115	0.113	0.111
1.4	0.197	0.148	0.128	0.117	0.110	0.106	0.102	0.100	0.098	0.096
1.5	0.187	0.136	0.115	0.104	0.097	0.092	0.089	0.086	0.084	0.082
1.6	0.178	0.125	0.104	0.092	0.085	0.080	0.077	0.074	0.072	0.070
1.7	0.169	0.116	0.094	0.082	0.075	0.070	0.066	0.064	0.062	0.060
1.8	0.161	0.107	0.085	0.073	0.066	0.061	0.057	0.055	0.053	0.051
1.9	0.154	0.099	0.077	0.065	0.058	0.053	0.050	0.047	0.045	0.043
2.0	0.148	0.092	0.070	0.058	0.051	0.046	0.043	0.040	0.038	0.037
2.1	0.141	0.085	0.063	0.052	0.045	0.040	0.037	0.034	0.033	0.031
2.2	0.136	0.079	0.058	0.046	0.040	0.035	0.032	0.029	0.028	0.026
2.3	0.131	0.074	0.052	0.041	0.035	0.031	0.027	0.025	0.023	0.022
2.4	0.126	0.069	0.048	0.037	0.031	0.027	0.024	0.022	0.020	0.019
2.5	0.121	0.065	0.044	0.033	0.027	0.023	0.020	0.018	0.017	0.016
2.6	0.117	0.061	0.040	0.030	0.024	0.020	0.018	0.016	0.014	0.013
2.7	0.113	0.057	0.037	0.027	0.021	0.018	0.015	0.014	0.012	0.011
2.8	0.109	0.054	0.034	0.024	0.019	0.016	0.013	0.012	0.010	0.009
2.9	0.106	0.051	0.031	0.022	0.017	0.014	0.011	0.010	0.009	0.008
3.0	0.102	0.048	0.029	0.020	0.015	0.012	0.010	0.009	0.007	0.007
3.1	0.099	0.045	0.027	0.018	0.013	0.011	0.009	0.007	0.006	0.006
3.2	0.096	0.043	0.025	0.016	0.012	0.009	0.008	0.006	0.005	0.005
3.3	0.094	0.040	0.023	0.015	0.011	0.008	0.007	0.005	0.005	0.004
3.4	0.091	0.038	0.021	0.014	0.010	0.007	0.006	0.005	0.004	0.003
3.5	0.089	0.036	0.020	0.012	0.009	0.006	0.005	0.004	0.003	0.003
3.6	0.086	0.035	0.018	0.011	0.008	0.006	0.004	0.003	0.003	0.002
3.7	0.084	0.033	0.017	0.010	0.007	0.005	0.004	0.003	0.002	0.002
3.8	0.082	0.031	0.016	0.010	0.006	0.004	0.003	0.003	0.002	0.002
3.9	0.080	0.030	0.015	0.009	0.006	0.004	0.003	0.002	0.002	0.001
4.0	0.078	0.029	0.014	0.008	0.005	0.004	0.003	0.002	0.002	0.001
4.1	0.076	0.027	0.013	0.007	0.005	0.003	0.002	0.002	0.001	0.001
4.2	0.074	0.026	0.012	0.007	0.004	0.003	0.002	0.001	0.001	0.001
4.3	0.073	0.025	0.012	0.006	0.004	0.003	0.002	0.001	0.001	0.001

Tail Areas for t Curves

$t^* \backslash df$	11	12	13	14	15	16	17	18	19	20
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461
0.2	0.423	0.422	0.422	0.422	0.422	0.422	0.422	0.422	0.422	0.422
0.3	0.385	0.385	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384
0.4	0.348	0.348	0.348	0.348	0.347	0.347	0.347	0.347	0.347	0.347
0.5	0.313	0.313	0.313	0.312	0.312	0.312	0.312	0.312	0.311	0.311
0.6	0.280	0.280	0.279	0.279	0.279	0.278	0.278	0.278	0.278	0.278
0.7	0.249	0.249	0.248	0.248	0.247	0.247	0.247	0.246	0.246	0.246
0.8	0.220	0.220	0.219	0.219	0.218	0.218	0.217	0.217	0.217	0.217
0.9	0.194	0.193	0.192	0.192	0.191	0.191	0.190	0.190	0.190	0.189
1.0	0.169	0.169	0.168	0.167	0.167	0.166	0.166	0.165	0.165	0.165
1.1	0.147	0.146	0.146	0.145	0.144	0.144	0.143	0.143	0.143	0.142
1.2	0.128	0.127	0.126	0.125	0.124	0.124	0.123	0.123	0.122	0.122
1.3	0.110	0.109	0.108	0.107	0.107	0.106	0.105	0.105	0.105	0.104
1.4	0.095	0.093	0.092	0.092	0.091	0.090	0.090	0.089	0.089	0.088
1.5	0.081	0.080	0.079	0.078	0.077	0.077	0.076	0.075	0.075	0.075
1.6	0.069	0.068	0.067	0.066	0.065	0.065	0.064	0.064	0.063	0.063
1.7	0.059	0.057	0.056	0.056	0.055	0.054	0.054	0.053	0.053	0.052
1.8	0.050	0.049	0.048	0.047	0.046	0.045	0.045	0.044	0.044	0.043
1.9	0.042	0.041	0.040	0.039	0.038	0.038	0.037	0.037	0.036	0.036
2.0	0.035	0.034	0.033	0.033	0.032	0.031	0.031	0.030	0.030	0.030
2.1	0.030	0.029	0.028	0.027	0.027	0.026	0.025	0.025	0.025	0.024
2.2	0.025	0.024	0.023	0.023	0.022	0.021	0.021	0.021	0.020	0.020
2.3	0.021	0.020	0.019	0.019	0.018	0.018	0.017	0.017	0.016	0.016
2.4	0.018	0.017	0.016	0.015	0.015	0.014	0.014	0.014	0.013	0.013
2.5	0.015	0.014	0.013	0.013	0.012	0.012	0.011	0.011	0.011	0.011
2.6	0.012	0.012	0.011	0.010	0.010	0.010	0.009	0.009	0.009	0.009
2.7	0.010	0.010	0.009	0.009	0.008	0.008	0.008	0.007	0.007	0.007
2.8	0.009	0.008	0.008	0.007	0.007	0.006	0.006	0.006	0.006	0.006
2.9	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.005	0.005	0.004
3.0	0.006	0.006	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004
3.1	0.005	0.005	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.003
3.2	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002
3.3	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002
3.4	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
3.5	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
3.6	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.7	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.8	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.9	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
4.0	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
4.1	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
4.2	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.3	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Tail Areas for t Curves

$t^* \backslash df$	21	22	23	24	25	26	27	28	29	30
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461
0.2	0.422	0.422	0.422	0.422	0.422	0.422	0.421	0.421	0.421	0.421
0.3	0.384	0.383	0.383	0.383	0.383	0.383	0.383	0.383	0.383	0.383
0.4	0.347	0.347	0.346	0.346	0.346	0.346	0.346	0.346	0.346	0.346
0.5	0.311	0.311	0.311	0.311	0.311	0.311	0.311	0.310	0.310	0.310
0.6	0.277	0.277	0.277	0.277	0.277	0.277	0.277	0.277	0.277	0.277
0.7	0.246	0.246	0.245	0.245	0.245	0.245	0.245	0.245	0.245	0.245
0.8	0.216	0.216	0.216	0.216	0.216	0.215	0.215	0.215	0.215	0.215
0.9	0.189	0.189	0.189	0.189	0.188	0.188	0.188	0.188	0.188	0.188
1.0	0.164	0.164	0.164	0.164	0.163	0.163	0.163	0.163	0.163	0.163
1.1	0.142	0.142	0.141	0.141	0.141	0.141	0.141	0.140	0.140	0.140
1.2	0.122	0.121	0.121	0.121	0.121	0.120	0.120	0.120	0.120	0.120
1.3	0.104	0.104	0.103	0.103	0.103	0.103	0.102	0.102	0.102	0.102
1.4	0.088	0.088	0.087	0.087	0.087	0.087	0.086	0.086	0.086	0.086
1.5	0.074	0.074	0.074	0.073	0.073	0.073	0.073	0.072	0.072	0.072
1.6	0.062	0.062	0.062	0.061	0.061	0.061	0.061	0.060	0.060	0.060
1.7	0.052	0.052	0.051	0.051	0.051	0.051	0.050	0.050	0.050	0.050
1.8	0.043	0.043	0.042	0.042	0.042	0.042	0.042	0.041	0.041	0.041
1.9	0.036	0.035	0.035	0.035	0.035	0.034	0.034	0.034	0.034	0.034
2.0	0.029	0.029	0.029	0.028	0.028	0.028	0.028	0.028	0.027	0.027
2.1	0.024	0.024	0.023	0.023	0.023	0.023	0.023	0.022	0.022	0.022
2.2	0.020	0.019	0.019	0.019	0.019	0.018	0.018	0.018	0.018	0.018
2.3	0.016	0.016	0.015	0.015	0.015	0.015	0.015	0.015	0.014	0.014
2.4	0.013	0.013	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.011
2.5	0.010	0.010	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009
2.6	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007
2.7	0.007	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
2.8	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.004
2.9	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003
3.0	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
3.1	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.2	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.3	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
3.4	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.5	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.6	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.7	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
3.8	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Tail Areas for t Curves

$t^* \backslash df$	40	50	60	70	80	90	100	110	120	$\infty = z$
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460	0.460
0.2	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421
0.3	0.383	0.383	0.383	0.383	0.382	0.382	0.382	0.382	0.382	0.382
0.4	0.346	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.345
0.5	0.310	0.310	0.309	0.309	0.309	0.309	0.309	0.309	0.309	0.309
0.6	0.276	0.276	0.275	0.275	0.275	0.275	0.275	0.275	0.275	0.274
0.7	0.244	0.244	0.243	0.243	0.243	0.243	0.243	0.243	0.243	0.242
0.8	0.214	0.214	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.212
0.9	0.187	0.186	0.186	0.186	0.185	0.185	0.185	0.185	0.185	0.184
1.0	0.162	0.161	0.161	0.160	0.160	0.160	0.160	0.160	0.160	0.159
1.1	0.139	0.138	0.138	0.138	0.137	0.137	0.137	0.137	0.137	0.136
1.2	0.119	0.118	0.117	0.117	0.117	0.117	0.116	0.116	0.116	0.115
1.3	0.101	0.100	0.099	0.099	0.099	0.098	0.098	0.098	0.098	0.097
1.4	0.085	0.084	0.083	0.083	0.083	0.082	0.082	0.082	0.082	0.081
1.5	0.071	0.070	0.069	0.069	0.069	0.069	0.068	0.068	0.068	0.067
1.6	0.059	0.058	0.057	0.057	0.057	0.057	0.056	0.056	0.056	0.055
1.7	0.048	0.048	0.047	0.047	0.047	0.046	0.046	0.046	0.046	0.045
1.8	0.040	0.039	0.038	0.038	0.038	0.038	0.037	0.037	0.037	0.036
1.9	0.032	0.032	0.031	0.031	0.031	0.030	0.030	0.030	0.030	0.029
2.0	0.026	0.025	0.025	0.025	0.024	0.024	0.024	0.024	0.024	0.023
2.1	0.021	0.020	0.020	0.020	0.019	0.019	0.019	0.019	0.019	0.018
2.2	0.017	0.016	0.016	0.016	0.015	0.015	0.015	0.015	0.015	0.014
2.3	0.013	0.013	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.011
2.4	0.011	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.009	0.008
2.5	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.007	0.006
2.6	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.7	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003
2.8	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.9	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
3.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
3.1	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
3.4	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
3.5	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000