UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT113 Calculus II Exam 2A November 1, 2024

Print Name:

UAlbany Email:

Instructors Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	8	
2	8	
3	6	
4	10	
5	10	
6	8	
Total	50	

YOUR SIGNATURE:

(1) (8 points) Find the arc length of the following function from $x = 0$ to $x = 9$

$$f(x) = 2x^{3/2} - 5$$

YOUR SIGNATURE:

(2) (8 points) Use integration by parts to evaluate the following integral. You must clearly declare your u and dv for full credit.

$$\int_0^{\pi/3} (3x - 4)\cos(3x)\,dx$$

YOUR SIGNATURE:

(3) (6 points) Give the partial fraction decomposition for $f(x) = \dfrac{x^2+8}{x(x+1)^2}$. You **DO NOT** need to integrate.

YOUR SIGNATURE:

(4) Evaluate the following integrals using known trigonometric methods or substitution. (5 points each)

(a) $\int \sin^3(x)\cos^4(x)\,dx$

(b) $\int \frac{3}{\sqrt{x^2-16}}\,dx$

YOUR SIGNATURE:

(5) In each of the following parts determine whether the given improper integral converges or diverges. If it converges, compute it. If it diverges, show the divergent limit or use the Comparison Theorem to explain why. (5 points each)

(a) $\displaystyle\int_0^\infty \frac{2x}{x^2+12}\,dx$

(b) $\displaystyle\int_{-3}^{6} \frac{1}{2\sqrt{x+3}}\,dx$

YOUR SIGNATURE:

(6) Determine whether the following the sequences converge or diverge. If the sequence converges, then compute its limit. (4 points each)

(a) $\left\{\dfrac{2n^5+n^2-9}{3n^5-2n^4+2n^3+1}\right\}$

(b) $\left\{(-1)^{n+1}\left(\dfrac{2n-1}{3n}\right)\right\}$

YOUR SIGNATURE:

Integration by Parts:

$$\int udv = uv - \int vdu \qquad \text{or} \qquad \int uv'dx = uv - \int vu'dx$$

Useful Integrals for Comparison:

$\int_1^\infty \frac{1}{x^p}\,dx$ converges for $p > 1$ and diverges for $p \leq 1$.
$\int_0^1 \frac{1}{x^p}\,dx$ converges for $p < 1$ and diverges for $p \geq 1$.
$\int_0^\infty e^{-ax}\,dx$ converges for $a > 0$.

Trigonometric Identities: $\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta)$

Differentiation formulas

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C,\ (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int a^x\,dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\,dx = x\ln x - x + C$
5. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$
6. $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$
7. $\int \tan ax\,dx = -\frac{1}{a}\ln|\cos ax| + C$
8. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$