AMAT112 Calculus I — Final Exam B — Fall 2024

Print Name:

UAlbany Email:

Please indicate your lecture section with a check mark (✓) in the leftmost column.

✓	Class No	Professor	Time and location
	3885	Lynn Greene	MWF 8:00-9:15AM AS0014
	3886	Riley Decker	MWF 8:00-9:15AM ES0147
	3887	Effie Shani	MWF 11:40-12:55PM AS0014
	3888	Sam Spellman	MWF 4:30-5:45PM TA0118
	3889	Marquia Williams	MWF 11:40-12:55PM ES0124
	3890	John Habib	MWF 1:10-2:25PM HU0133
	3926	Seth Hulbert	MWF 1:10-2:25PM HU0129
	4047	Zhijian (Lamber) Guo	MWF 4:30-5:45PM HU0133
	4453	Zhijian (Lamber) Guo	MWF 3:00-4:15PM HU0132
	4454	Sam Spellman	MWF 3:00-4:15PM TA0118
	4786	Mira Iskander	MTTh 4:30-5:50PM HU0124
	5025	Catherine Hall	TWTh 4:30-5:50PM ES0139
	6596	Seth Hulbert	MWF 8:00-9:15AM SS0116
	7039	Effie Shani	MWF 8:00-9:15AM HU0128

Directions: You have **120 minutes**. Show all necessary work as neatly and clearly as possible. No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points	Problem	Possible	Points
1	10		6	8	
2	9		7	16	
3	9		8	8	
4	12		9	10	
5	8		10**	8	
Total (Out of 90) =					

**Optional Extra Credit Problem

(1) A certain painkiller has a dosage, D, that depends on the patient's weight, w. The function $D = f(w)$ represents this relationship, where D is measured in milligrams (mg) and w is measured in pounds (lbs).

(a) (2 Points) Which of the following best explains the meaning of the statement $f(160) = 125$ in the context of this painkiller? Circle your answer.

(i) A patient who weighs 125 lbs should receive 160 mg of the painkiller.

(ii) A patient who weighs 160 lbs should receive 125 mg of the painkiller.

(iii) The painkiller dosage increases by 125 mg for every 160 lbs of weight.

(iv) The painkiller dosage increases by 160 mg for every 125 lbs of weight.

(v) The maximum dosage of the painkiller is 125 mg for a patient weighing 160 lbs.

(b) (2 Points) Which of the following best explains the meaning of the statement $f'(160) = 2$ in the context of this painkiller?

(i) A patient who weighs 160 lbs should receive 2 mg of the painkiller.

(ii) The dosage of the painkiller increases by 2 mg for every 160 lbs of weight.

(iii) For a patient weighing 160 lbs, the dosage increases by 2 mg for every additional pound of weight.

(iv) For a patient weighing 160 lbs, the dosage increases by 2 lbs for every additional milligram of the painkiller.

(v) A patient who weighs 162 lbs should receive an additional 2 mg of the painkiller compared to a patient who weighs 160 lbs.

(c) (6 Points) Based on the information given in parts (a) and (b), use a linear approximation to estimate the value of $f(155)$. Show your work.

(2) Compute the derivatives of each of the following functions. You **do not** have to simplify your final answer.

(a) (3 Points) $f(x) = 3\pi - 4x^2 + 4e^x$

(b) (3 Points) $h(x) = \ln(3x^5 - 2x + 1)$

(c) (3 Points) $k(x) = \dfrac{1 - \tan(x)}{x^2}$

(3) (a) (5 Points) Consider the curve

$$xy^2 + \sin(y) = 1$$

Find an expression for $\dfrac{dy}{dx}$.

(b) (4 Points) Use the table:

x	-2	-1	0	1	2	3
$g(x)$	5	-2	2	-1	3	1
$g'(x)$	-1	5	3	1	-2	2

to evaluate $\dfrac{d}{dx}\left(g^{-1}(x)\right)\Big|_{x=-2}$.

(4) (3 Points Each) Compute the value of each limit.

(a) $\displaystyle\lim_{x\to\infty} \frac{17x^3 - 3}{34x^3 + 1}$

(b) $\displaystyle\lim_{x\to\frac{1}{2}^-} \frac{|2x - 1|}{2x - 1}$

(c) $\displaystyle\lim_{x\to\infty} \frac{2e^{-x} + 1}{e^{-x} + 2}$

(d) $\displaystyle\lim_{x\to 0} \frac{x}{\cos(2x)}$

(5) (8 Points) Use L'Hopital's rule to calculate

$$\lim_{x \to 0} \frac{\ln(1-2x)}{\sin(4x)}.$$

(6) (a) (4 Points) Let $G(x)$ be a piecewise function defined by

$$G(x) = \begin{cases} 3e^x & \text{if } x < 1 \\ \\ 2x + b & \text{if } x \geq 1. \end{cases}$$

The function G is continuous everywhere, if $b =$

(b) (4 Points) If $q(x) = \dfrac{4}{x} + 2x$, what input c satisfies the conclusion of the Mean Value Theorem applied to $q(x)$ on the interval $[1, 4]$?

(7) Consider the function $g(x)$ and its derivatives

$$g(x) = 3x^{2/3} - 2x, \qquad g'(x) = \frac{2}{x^{1/3}} - 2, \qquad g''(x) = -\frac{2}{3x^{4/3}}.$$

The critical numbers of $g(x)$ are $x = 0$ and $x = 1$. Fill in the blanks in each question part below. If there is no value or interval, write DNE.

(a) (5 Points) $g(x)$ is increasing on the interval(s) ______________________________

and decreasing on the interval(s) ______________________________.

(b) (3 Points) $g(x)$ has a local minimum at $x =$______________

and a local maximum at $x =$______________.

(Problem 7 Continued)

$$g(x) = 3x^{2/3} - 2x, \qquad g'(x) = \frac{2}{x^{1/3}} - 2, \qquad g''(x) = -\frac{2}{3x^{4/3}}.$$

The critical numbers of $g(x)$ *are* $x = 0$ *and* $x = 1$.

(c) (4 Points) $g(x)$ is concave down on the interval(s) ____________________

and concave up on the interval(s) ____________________

(d) Suppose the function $g(x) = 3x^{2/3} - 2x$ is defined on the closed interval $[0, 8]$.

(i) (4 Points) The absolute minimum value of g is __________.

and the absolute maximum value of g is __________.

(8) (8 Points) A rectangle has a fixed perimeter of 40 cm.

(a) Find a function that models the area of the rectangle in terms of a single variable.

(b) Determine the dimensions of the rectangle that maximize its area.

(c) What is the maximum area?

(9) (4 Points Each) Evaluate the indefinite integrals.

(a) $\displaystyle\int \frac{5x^3 + x^2 - 2}{x^3}\, dx$

(b) $\displaystyle\int \big(3e^x - 2\sec^2(x)\big)\, dx$

Optional Extra Credit Problems

(10) (a) (4 Points) Use geometry to evaluate the integral below.

$$\int_0^2 \sqrt{4-x^2}dx$$

(b) (4 Points) A table of values of a function f is given below.

x	0	9	18	27	36
$f(x)$	-10	15	-5	20	5

Find a left-endpoint approximation with $n = 4$ for $\int_0^{36} f(x)dx$.

Formulas you might find useful

- **The derivative of a function**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Some rules of differentiation**

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

- **The equation of the tangent line to a function f for $x = a$ is given by**

$$y = f(a) + f'(a)(x - a)$$

- **The derivative of the inverse function f^{-1} at $x = a$ is given by**

$$\frac{d}{dx}\left(f^{-1}(x)\right)\bigg|_{x=a} = \frac{1}{f'(f^{-1}(a))}.$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$