UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT112 Calculus I — Exam 3A — Fall 2024

Print Name:
UAlbany Email:
Instructor's Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	10	
2	5	
3	15	
4	12	
5	10	
6	10	
7**	8	
Total	62	

**Optional Extra Credit Problems

YOUR SIGNATURE:

(Similar to Practice Assessment 11)

(1) (a) (5 Points) Use a linear approximation to estimate the value of $f(9)$ if $f(12) = 6$ and $f'(12) = -2$.

(b) (5 Points) Let $y = (x^2 - 1)^2$. Find the differential dy when $x = -1$ and $dx = 0.2$.

(Similar to Practice Assessment 12)

(2) (5 Points) If $h(x) = 3\sin(4x)$, what input c satisfies the conclusion of the Mean Value Theorem applied to $h(x)$ on the interval $\left[0, \frac{\pi}{4}\right]$?

YOUR SIGNATURE:

(Similar to Practice Assessments 13 and 14)

(3) (15 Points) Consider the function $f(x) = -2x^3 + 6x^2 - 3$.

Fill in the blanks below.

(a) The critical number(s) of f is (are)_____.

(b) f is increasing on the interval(s)_____.

(c) f has a local minimum point at $x =$_____and a local maximum point at $x =$_____.

YOUR SIGNATURE:

(Problem 3 Continued) Consider the function $f(x) = -2x^3 + 6x^2 - 3$.

(d) The absolute maximum value of f on the interval $[-1, 1]$ is _____.

(e) f is concave up on the interval _____.

(f) f has an inflection point at $x =$_____.

YOUR SIGNATURE:

(Similar to Practice Assessment 15)

(4) (4 Points Each) For each of the following limits determine which indeterminate form the expression corresponds to, then calculate the limit.

(a) $\lim_{x \to 1} \dfrac{e^x - e}{1 - x}$

(b) $\lim_{x \to \infty} \dfrac{\ln(3x)}{3x}$

(c) $\lim_{x \to \infty} (\ln(3x + 2) - \ln(x + 5))$

YOUR SIGNATURE:

(Similar to Practice Assessment 16)

(5) (10 Points) A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume? *Clearly show all your work.*

(Similar to Practice Assessment 17)

(6) (5 Points Each) Evaluate the indefinite integrals.

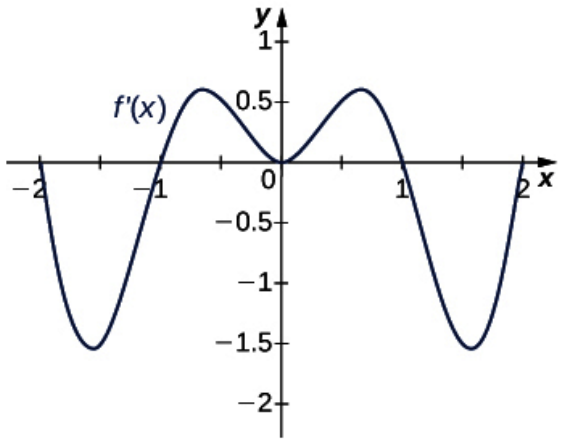
(a) $\displaystyle\int \frac{2+\sqrt{x}+x}{\sqrt{x}}\,dx$

(b) $\displaystyle\int (3e^x - 7x + \pi\cos(x) + 2)\,dx$

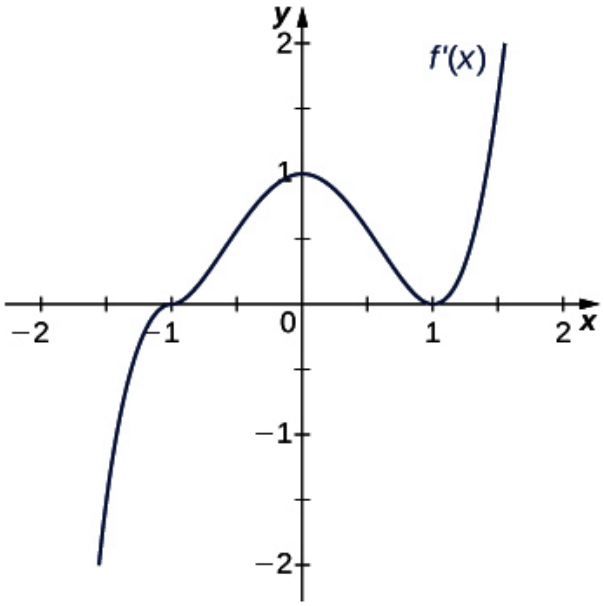
YOUR SIGNATURE:

(Similar to Practice Assessment 14)**(Optional Extra Credit Problems)**

(7) (a) Analyze the graph of f' below, then list all intervals where f is increasing or decreasing,



(b) Analyze the graph of f' below, then list all the intervals where f is concave up or concave down and all inflections points.



YOUR SIGNATURE:

Formulas you might find useful

- **The derivative of a function**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Some rules of differentiation**

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- **The equation of the tangent line to a function f for $x = a$ is given by**

$$y = f(a) + f'(a)(x - a)$$

- **The derivative of the inverse function f^{-1} at $x = a$ is given by**

$$\frac{d}{dx}\left(f^{-1}(x)\right)\Big|_{x=a} = \frac{1}{f'(f^{-1}(a))}.$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$