AMAT112 Calculus I   Exam 2A   Fall 2024

Print Name:

UAlbany Email:

Instructor's Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

(Similar to Practice Assessment 6 Problems 1,2,3)

(1) Compute the derivatives of each of the following functions. You **do not** have to simplify your final answer.

(a) (3 Points) $h(x) = 3e^x - 4x^2 + e^\pi$

(b) (3 Points) $f(x) = (4 - 2x^3)(x^{\frac{1}{2}} - 1)$

(c) (4 Points) $k(x) = \dfrac{1 + \sin(x)}{1 + \cos(x)}$

(Similar to Practice Assessment 8 Problems 1,2)

(2) (a) (4 Points) Calculate $\dfrac{d}{dx}\left(f^{-1}(x)\right)\Big|_{x=a}$ for the function:

$$f(x) = x - \sqrt{x}, \qquad a = 2.$$

(Hint: $f(4) = 2$.)

(b) (3 Points Each) Find the derivative of each of the following:

(i) $G(x) = \tan^{-1}(1 + x)$

(ii) $u(t) = \sin^{-1}(e^{5t})$

(Similar to Practice Assessment 6 Problem 4) (5 Points Each)

(3) Suppose f and g are differentiable functions with values shown in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
5	2	1	3	0

(a) Find the exact value of $h'(5)$ if $h(x) = \dfrac{1 - 3f(x)}{g(x)}$. Show all your work and **simplify your final answer**.

(b) Find the exact value of $w'(5)$ if $w(x) = \sqrt{f(x) \cdot g(x)}$. Show all your work and **simplify your final answer**.

(Similar to Practice Assessment 7 Problems 3, 4) (5 Points Each)

(4) Assume that y is a differentiable function of x and that
$$2x + \cos(x - y) + y^2 = \pi.$$

(a) Compute $\dfrac{dy}{dx}$.

(b) Find the equation of the tangent line to the curve given by $2x + \cos(x - y) + y^2 = \pi$ at the point $\left(\dfrac{\pi}{2}, 0\right)$.

(Similar to Practice Assessment 10 Problems 1c, 2a) (5 Points Each)

(5) Use logarithmic differentiation and/or the properties of logarithms to compute the derivative, y', of the following functions.

(a)
$$y = x^{\sqrt{x}}$$

(b)
$$y = \ln\left(\frac{(x+1)^3(x-3)^7}{(x+2)^5}\right)$$

Formulas you might find useful

- **The derivative of a function**
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Some rules of differentiation**
$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- **The equation of the tangent line to a function f for $x = a$ is given by**
$$y = f(a) + f'(a)(x - a)$$

- **The derivative of the inverse function f^{-1} at $x = a$ is given by**
$$\left.\frac{d}{dx}\left(f^{-1}(x)\right)\right|_{x=a} = \frac{1}{f'(f^{-1}(a))}.$$

- **Differentiation formulas**

$\dfrac{d}{dx}(x^n) = nx^{n-1}$	$\dfrac{d}{dx}(e^x) = e^x$	$\dfrac{d}{dx}(a^x) = (\ln a)a^x$
$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$	$\dfrac{d}{dx}(\sin(x)) = \cos x$	$\dfrac{d}{dx}(\cos(x)) = -\sin x$
	$\dfrac{d}{dx}(\tan(x)) = \sec^2 x$	$\dfrac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\dfrac{d}{dx}(\sec(x)) = \sec x \tan x$	$\dfrac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\dfrac{d}{dx}(\sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$	$\dfrac{d}{dx}(\cos^{-1}(x)) = \dfrac{-1}{\sqrt{1-x^2}}$	$\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$