UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT 108 Elementary Statistics
Fall 2024

Final Exam
Version A

Print Name: **Answer Key**

UAlbany Email:

Directions: You have **120 minutes** to answer the following questions. For all multiple-choice questions, select **one** answer from among the choices given. No explanation is required to be shown, and no partial credit will be given. Make sure to **completely** fill in the circle corresponding to your chosen answer. For all free-response questions, you **must** show all necessary work to receive full credit. An answer with no work, even if correct, will not receive full credit. Please circle or box your final answer. All work, if needed, is to be rounded to **at least five** decimal places.

No notes, textbooks, mobile phones, or other aids are allowed. Only scientific calculators are allowed. **Do not detach these pages.**

Please indicate your section with a check mark (✓) in the left-most column.

✓	Section	Instructor Name	Meeting Time	Meeting Days	Meeting Location
	1724	Douglas Rosenberg	3:00PM	T/TH	ES 140
	1725	James Lamatina	12:00PM	T/TH	LC 25
	1726	John Racquet	3:00PM	M/W	LC 2
	3414	Chris Lange	4:30PM	T/TH	HU 132
	3807	Tung Lam	11:40AM	M/W	HU 129
	3808	Luciano Medina	10:30AM	T/TH	ES 144
	3809	James Lamatina	9:00AM	T/TH	HU 124
	4551	Tung Lam	8:00AM	M/W	ES 140
	4803	Peter Young	9:00AM	T/TH	ES 139
	4960	Peter Young	12:00PM	T/TH	ES 140
	5574	Chris Lange	3:00PM	T/TH	SS 133

Exam Scoring:

Questions	Possible Points	Points Earned
1-10	10	
11	5	
12	8	
13	9	
14	9	
15	7	
16	13	
17	10	
Total Points	71	
Percentage		

Questions 1-4 refer to the following: Suppose A, B, D, and E are events with $P(A) = 0.43$, $P(B) = 0.14$, $P(D) = 0.27$, and $P(E) = 0.13$.

1. If A and B are disjoint, find $P(A \cup B)$. (1 pt.)

① 0.7
② 0.57 (marked)
③ 0.06020
④ 0.56
⑤ None of the previous options.

2. If A and B are independent, find $P(A \cap B)$. (1 pt.)

① 0.7
② 0.57
③ 0.06020 (marked)
④ 0.56
⑤ None of the previous options.

3. Given that $P(A \cap D) = 0.1975$, find $P(A|D)$. (1 pt.)

① 0.73148 (marked)
② 0.27
③ 0.43
④ 1.36709
⑤ 0.45930

4. Find $P(E^C)$. (1 pt.)

① 0.84
② 0.18018
③ 0.1690
④ 0.87 (marked)
⑤ None of the previous options.

5. Suppose X is a discrete random variable that has possible values 5, 8, and 13. Let $P(X = 5) = 0.2$ and $P(X = 8) = 0.31$. Find $P(X = 13)$. (1 pt.)

① 0.49 (marked)
② 0.31
③ 0.51
④ 0.2
⑤ None of the previous options.

Questions 6 and 7 refer to the following: A bin contains ten golf balls, eight baseballs, and four tennis balls. Mr. Rosenberg selects two balls from the bin at random.

6. Assume sampling is done with replacement. Find the probability that the second ball picked is a golf ball, given that the first ball picked is a baseball. (1 pt.)

① 0.45455
② 0.47619
③ 0.36364
④ 0.38095
⑤ None of the previous options.

7. Assume sampling is done without replacement. Find the probability that the second ball picked is a golf ball, given that the first ball picked is a baseball. (1 pt.)

① 0.45455
② 0.47619
③ 0.36364
④ 0.38095
⑤ None of the previous options.

Questions 8 and 9 refer to the following: Assume Z has a standard normal distribution.

8. Find the closest value of c so that $P(Z \leq c) \approx 0.1056$. (1 pt.)

① -1.25
② -1.24
③ -1.26
④ -2.16
⑤ -2.15

9. Correct to four decimal places, find $P(-2.96 \leq Z \leq 1.73)$. (1 pt.)

① 0.9597
② 0.0015
③ 0.9567
④ 0.9582
⑤ None of the previous options.

10. Suppose $n = 30$ and p are such that all requirements for both confidence intervals are met. Find an appropriate 97% t^* critical number. (1 pt.)

① 2.170
② 2.278
③ 2.282
④ 2.150
⑤ 2.462

11. Suppose Y has a uniform distribution on the interval $(2, 17)$.
   (a) Find the height of the density curve. (2 pts.)

$$f(x) = \frac{1}{17-2} = \frac{1}{15} \quad \longrightarrow \quad \text{height} = \frac{1}{15} \ \boxed{+2} \tag{1}$$

Note. Award one point for the numerator and one point for the denominator.

   (b) Find the probability that Y is greater than or equal to 13. Round your answer to *five* decimal places. (3 pts.)

$$P(Y \geq 13) = \frac{17-13}{15} \tag{2}$$

$$\approx 0.26667 \tag{3}$$

- $\boxed{+2}$ for (2) (one point for numerator and one point for denominator)
- $\boxed{+1}$ for (3)

Note. If the student obtains 0.86667 as their answer and has 13 as the numerator, deduct one point.

12. The discrete random variable X has probability distribution given in the table below:

X	3	12	18
$p(X)$	0.18	0.27	0.55

   (a) Find the mean value of X. Do not round your answer. (3 pts.)

$$\mu_X = E(X) = 3(0.18) + 12(0.27) + 18(0.55) \tag{4}$$

$$= 13.68 \ \boxed{+1} \tag{5}$$

- $\boxed{+2}$ for (4) (one point for all products and one point for adding products together)
- $\boxed{-2}$ if the student shows

$$\mu_X = E(X) = \frac{3+12+18}{3} = 11. \tag{6}$$

- $\boxed{-3}$ if the student shows

$$\mu_X = E(X) = \frac{0.18+0.27+0.55}{3} \approx 0.33333 \tag{7}$$

   (b) Find the standard deviation of X. Round your answer to three decimal places. (5 pts.)

$$\sigma_X = \sqrt{.18(3-13.68)^2 + .27(12-13.68)^2 + .55(18-13.68)^2} \tag{8}$$

$$\approx 5.618 \tag{9}$$

- $\boxed{+4}$ for (8) (one point for squared deviations from mean, one point for products, one point for adding products, and one point for taking a square root)
- $\boxed{+1}$ for (9)

Note. Some students may show:

$$E(X^2) = 3^2(.18) + 12^2(.27) + 18^2(.55) \ \boxed{+1} \tag{10}$$

$$= 218.7 \ \boxed{+1} \tag{11}$$

$$\sigma_X = \sqrt{218.7 - (13.68)^2} \tag{12}$$

$$\approx 5.618 \ \boxed{+1} \tag{13}$$

- $\boxed{+2}$ for (12) (one point for square root and one point for contents of square root)

13. Suppose it is a widely-held belief that the weights of all dogs (in pounds) follow a normal distribution with mean 33 and standard deviation 2.2. Mr. Lam selects 175 dogs at random and records their weight in pounds.

(a) Is the distribution of $\overline{X}$ normal? (1 pt.)

① Yes, because the population has a normal distribution with mean 33 pounds and standard deviation 2.2 pounds.

② No, but the distribution is approximately normal because the sample size is large enough.

③ No, and we cannot conclude anything about the distribution of $\overline{X}$.

(b) Compute the probability that the average weight of the 175 dogs Mr. Lam selected is less than or equal to 32.625 pounds. (4 pts.)

$$P(\overline{X} \le 32.625) = P\left(\frac{\overline{X} - 33}{\left(\frac{2.2}{\sqrt{175}}\right)} \le \frac{32.625 - 33}{\left(\frac{2.2}{\sqrt{175}}\right)}\right) \tag{14}$$

$$\approx P(Z \le -2.25) \tag{15}$$

$$\approx 0.0122 \tag{16}$$

- $\boxed{+2}$ for conversion from $\overline{X}$ to Z as shown in (14) (one point for numerator and one point for denominator)
- $\boxed{+1}$ for (15)
- $\boxed{+1}$ for (16)

(c) Compute the probability that the average weight of the 175 dogs Mr. Lam selected is between 32.625 pounds and 33.375 pounds inclusive. *Hint:* This is an example of a symmetric-limits problem. (4 pts.)

$$P(32.625 \le \overline{X} \le 33.375) = P\left(\frac{32.625 - 33}{\left(\frac{2.2}{\sqrt{175}}\right)} \le \frac{\overline{X} - 33}{\left(\frac{2.2}{\sqrt{175}}\right)} \le \frac{33.375 - 33}{\left(\frac{2.2}{\sqrt{175}}\right)}\right) \tag{17}$$

$$\approx P(-2.25 \le Z \le 2.25) \tag{18}$$

$$= P(Z \le 2.25) - P(Z \le -2.25) \tag{19}$$

$$\approx 0.9878 - 0.0122 \tag{20}$$

$$= 0.9756 \tag{21}$$

- $\boxed{+1}$ for conversion from $\overline{X}$ to Z as shown in (17)
- $\boxed{+1}$ for (18) (do not deduct this point if they only show z-scores)
- $\boxed{+1}$ for probabilities in (20) (no partial credit)
- $\boxed{+1}$ for (21)

Note. Some students may use the properties of symmetric-limits/balanced-limits problems by replacing (19) with $1 - 2P(Z \le -2.25)$ and (20) with $1 - 2(0.0122)$. Award one point each if these are shown. Do not deduct points if they only find the one z-score and the probability.

14. Mr. Young frequently experiences power outages at his home, prompting him to consider purchasing a permanent generator. An analysis of 35 randomly selected years of data reveals that 62% of those years had at least five power outages.

(a) Mr. Young wishes to construct a central 98% confidence interval for the proportion of all calendar years that have at least five power outages occur. Are all requirements met? (1 pt.)

① Yes, because the population of interest has a normal distribution.

② Yes, because the sample size is sufficiently large.

❸ Yes, because the success-failure conditions are met.

④ No, because none of the above apply.

(b) Construct a central 98% confidence interval for the proportion of all calendar years that have at least five power outages occur. Round *both* endpoints to *three* decimal places. (5 pts.)

$$.62 \pm 2.326\sqrt{\frac{.62(1-.62)}{35}} = \left(.62 - 2.326\sqrt{\frac{.62(1-.62)}{35}}, .62 + 2.326\sqrt{\frac{.62(1-.62)}{35}}\right) \tag{22}$$

$$\approx (.62 - 2.326(.08205), .62 + 2.326(.08205)) \tag{23}$$

$$\approx (.62 - .19084, .62 + .19084) \tag{24}$$

$$\approx (.429, .811) \tag{25}$$

- $\boxed{+3}$ for left-hand side of (22) (one point for point estimate, one point for square root, one point for fraction inside of square root, no partial credit)
- $\boxed{+2}$ for (25) (one point for each endpoint)

Note.

- $\boxed{-1}$ if the student gives (.811, .429) as their answer
- Equations (23), (24), and the right-hand side of (22) are shown for calculation purposes. Students only need to show the left-hand side of (22) and the answer in (25) to receive full credit.

(c) Find a conservative estimate for the sample size required to estimate the proportion of all calendar years that have at least five power outages occur to within 0.124 with 98% confidence. (3 pts.)

$$n = 0.5(1-0.5)\left(\frac{2.326}{0.124}\right)^2 \qquad n = \frac{1}{4}\left(\frac{2.326}{0.124}\right)^2 \tag{26}$$

$$\approx 87.966 \qquad \approx 87.966 \tag{27}$$

$$\uparrow 88 \qquad \uparrow 88 \tag{28}$$

- $\boxed{+2}$ for (26) (one point for recognizing a conservative estimate means $p = 0.5$ and one point for squaring inside fraction)
- $\boxed{+1}$ for (28)

Note. Either side can be shown for full credit.

15. The Math Department at SUNY Albany is investigating its toner usage for exams. To assess this, Mr. Lamatina and Dr. Racquet randomly selected 41 semesters and recorded the number of toner cartridges used. Their analysis found an average of 5.824 cartridges used per semester, with a standard deviation of 2.143 cartridges. The Department Chair previously believed that the average usage was 5.125 cartridges per semester. Mr. Lamatina and Dr. Racquet wish to know if there is enough evidence to conclude that the Chair's previous belief is an underestimate.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

① $H_0\colon \mu = 5.125$ vs. $H_1\colon \mu < 5.125$

② $H_0\colon \mu = 5.125$ vs. $H_1\colon \mu > 5.125$

③ $H_0\colon \mu = 5.125$ vs. $H_1\colon \mu \neq 5.125$

(b) Compute the t^* test statistic. Round your answer to *one* decimal place. (3 pts.)

$$t^* = \frac{5.824 - 5.125}{\left(\frac{2.143}{\sqrt{41}}\right)} \tag{29}$$

$$\approx 2.1 \tag{30}$$

- $\boxed{+2}$ for (29) (one point for numerator and one point for denominator)
- $\boxed{+1}$ for (30)

(c) The test statistic has a t distribution with how many degrees of freedom? (1 pt.)

① 39 ② 40 ③ 42 ④ 41

(d) Compute the *p-value* of the test, and express your answer to *three* decimal places. (1 pt.)

$$\text{P-value} = P_{H_0}(T \geq 2.1) \tag{31}$$

$$\approx 0.021 \boxed{+1} \tag{32}$$

Note. If the student forgets to put H_0 in the subscript outside the parentheses, do not deduct the point.

(e) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the Chair's previous belief is an underestimate.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the Chair's previous belief is an underestimate.

③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the Chair's previous belief is an underestimate.

④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the Chair's previous belief is an underestimate.

16. Over the years, Lipitor has helped many people lower their cholesterol and live healthier lives. However, some people have allergic reactions to it. Pfizer, the company that makes Lipitor, says they have found a way to make the drug that still lowers cholesterol but with fewer allergic reactions. Dr. Medina doesn't agree with their claim because of how their trial was done. He decides to do his own trial with people from all over the US, considering it a success if their cholesterol drops by more than 15% in six months. Summary statistics on the data he collected are shown below:

Drug	Lipitor	New Lipitor
Sample Size	3598	3284
Number of Successes	3470	3117
Success Rate	96.442%	94.915%

Dr. Medina wishes to use the 2-Sample Z Test for Equal Proportions to determine if there is enough evidence to conclude that the claim is incorrect. Use p_1 to denote the proportion of all individuals taking Lipitor who had their total cholesterol level improve by at least 15%, and use p_2 to denote the proportion of all individuals taking the new Lipitor who had their total cholesterol level improve by at least 15%.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

① $H_0: p_1 = p_2$ vs. $H_1: p_1 < p_2$

② $H_0: p_1 = p_2$ vs. $H_1: p_1 > p_2$

③ (selected) $H_0: p_1 = p_2$ vs. $H_1: p_1 \neq p_2$

(b) Compute the pooled sample proportion and round your answer to *five* decimal places. (3 pts.)

(33) $$\hat{p}_c = \frac{3470 + 3117}{3598 + 3284} \qquad \hat{p}_c = \frac{3598(0.96442) + 3284(0.94915)}{3598 + 3284}$$

(34) $$\approx 0.95713 \qquad \approx 0.95713$$

- $\boxed{+2}$ for (33) (one point for numerator and one point for denominator)
- $\boxed{+1}$ for (34)

(c) Compute the z^* test statistic. Round your answer to *two* decimal places, where appropriate. (5 pts.)

(35) $$z^* = \frac{.96442 - .94915}{\sqrt{\frac{.95713(1-.95713)}{3598} + \frac{.95713(1-.95713)}{3284}}} \qquad z^* = \frac{\frac{3470}{3598} - \frac{3117}{3284}}{\sqrt{\frac{.95713(1-.95713)}{3598} + \frac{.95713(1-.95713)}{3284}}}$$

(36) $$\approx \frac{.96442 - .94915}{.00489} \qquad \approx \frac{\frac{3470}{3598} - \frac{3117}{3284}}{.00489}$$

(37) $$\approx 3.12 \qquad \approx 3.12$$

- $\boxed{+1}$ for numerator in (35)
- $\boxed{+1}$ for square root in denominator of (35)
- $\boxed{+2}$ for fractions inside square root in (35) (one point for sample sizes and one point for $\hat{p}_c(1-\hat{p}_c)$, no partial credit)
- $\boxed{+1}$ for (37)

Note. Either side is acceptable.

Parts (d) and (e) are on the next page...

(d) Compute the *p-value* and express your answer to *four* decimal places. (3 pts.)

(38) $$P - value = 2P_{\mathrm{H}_0}(Z \leq -3.12)$$

(39) $$\approx 2(0.0009)$$

(40) $$= 0.0018$$

- +1 for (38) or something resembling finding the area of both tails
- +2 for (39) and (40) (one point each)

Note. If the student forgets to put H_0 in the subscript outside the parentheses, do not deduct any points.

(e) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the claim is incorrect.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the claim is incorrect.

③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the claim is incorrect.

④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the claim is incorrect.

17. Many years ago, Mr. Lange aspired to play professional golf at the highest level. However, he quickly realized that he wasn't talented enough to compete with the best in the world. To understand this further, Mr. Lange selects a random sample of his 18-hole golf scores from 2008-2024 and compares them with a random sample of 18-hole golf scores from PGA TOUR professionals in the same time frame. Summary statistics are shown below:

Golfer	PGA TOUR pros	Mr. Lange
Average	70.817	72.153
St. Deviation	2.361	3.582
Number of Rounds	352	111

Mr. Lange wishes to use Welch's T Test to determine if there is enough evidence to conclude that the PGA TOUR professionals are statistically better (score lower) than him. Use μ_1 to denote the population average 18-hole score returned by PGA TOUR professionals, and use μ_2 to denote the population average 18-hole score returned by Mr. Lange.

(a) Select the correct pair of statistical hypotheses. (1 pt.)

● 1 $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 < \mu_2$

② $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 > \mu_2$

③ $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$

(b) Compute the t^* test statistic. Round your answer to *one* decimal place. (5 pts.)

$$t^* = \frac{70.817 - 72.153}{\sqrt{\frac{2.361^2}{352} + \frac{3.582^2}{111}}} \tag{41}$$

$$\approx -3.7 \tag{42}$$

- $\boxed{+1}$ for numerator in (41)
- $\boxed{+2}$ for fractions in denominator of (41) (one point each, no partial credit)
- $\boxed{+1}$ for square root in denominator of (41)
- $\boxed{+1}$ for (42)

Parts (c), (d), and (e) are on the next page...

(c) Compute *df* for this test. (2 pts.)

(43) $$df = \min\{352, 111\} - 1$$
(44) $$= 111 - 1$$
(45) $$= 110$$

- +1 for either showing (43) or (44) (or both)
- +1 for (45)

(d) Compute the *p-value* and express your answer to *three* decimal places. (1 pt.)

(46) $$\text{P-value} = P_{H_0}(T \leq -3.7)$$
(47) $$= P_{H_0}(T \geq 3.7)$$
(48) $$\approx 0 \; \boxed{+1}$$

Note. If the student forgets to put H_0 in the subscript outside the parentheses, do not deduct the point.

(e) Which of the following is the correct conclusion? (1 pt.)

① Fail to reject H_0 at $\alpha = 0.05$. We do not have enough evidence to conclude that the PGA TOUR pros are statistically better than Mr. Lange.

② Reject H_0 at $\alpha = 0.05$, but fail to reject H_0 at $\alpha = 0.01$. We have a slight amount of evidence to conclude that the PGA TOUR pros are statistically better than Mr. Lange.

③ Reject H_0 at $\alpha = 0.01$, but fail to reject H_0 at $\alpha = 0.001$. We have a convincing amount of evidence to conclude that the PGA TOUR pros are statistically better than Mr. Lange.

④ Reject H_0 at $\alpha = 0.001$. We have an overwhelming amount of evidence to conclude that the PGA TOUR pros are statistically better than Mr. Lange.

Formula Sheet:

- Probability:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) \text{ if events } A \text{ and } B \text{ are disjoint}$$
$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B) \text{ if events } A \text{ and } B \text{ are independent}$$

- Complement probability (probability complement rule): $P(A^C) = 1 - P(A)$
- Conditional probability of event A given event B occurs:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- At least 1 Rule: $P(\text{at least 1 success in } n \text{ trials}) = 1 - P(\text{no successes in } n \text{ trials})$
- Finding the height of a uniform distribution:

$$\text{Height} = \frac{1}{\text{Base}} = \frac{1}{b-a}$$

- Probability (area) of a uniform distribution: Probability (or area) = Base · Height
- Mean of discrete random variable X with possible values $x_1, x_2, \ldots, x_n$:

$$\mu_X = E(X) = x_1 p(x_1) + x_2 p(x_2) + \cdots + x_n p(x_n)$$

- Standard deviation of a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$:

$$\sigma_X = \sqrt{(x_1 - \mu_X)^2 p(x_1) + (x_2 - \mu_X)^2 p(x_2) + \cdots + (x_n - \mu_X)^2 p(x_n)}$$
or
$$\sigma_X = \sqrt{E(X^2) - (E(X))^2}$$

- Mean and standard deviation for sampling distribution of $\overline{X}$:

$$\mu_{\overline{X}} = \mu \qquad \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

where μ = population mean and σ = population standard deviation

- Standardized variable (z-score) for $\overline{X}$ (when σ known):

$$Z = \frac{\overline{X} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

- Mean and standard deviation for sampling distribution of $\hat{p}$:

$$\mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

where p = population proportion of successes

- Sample proportion of successes:

$$\hat{p} = \frac{\text{number of successes in the sample}}{n}$$

- Standardized variable (z-score) for $\hat{p}$:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- Success/failure condition for the sampling distribution of $\hat{p}$ to be approximately normal:

$$np \geq 10 \qquad \text{and} \qquad n(1-p) \geq 10$$

- Confidence interval form:

$$\text{point estimate} \pm \text{margin of error}$$

  Margin of error is also the bound on the error of estimation
- Confidence interval for a population proportion:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Determining the sample size needed to be within M = margin of error under a certain confidence level:

$$n = p(1-p)\left(\frac{z^*}{M}\right)^2$$

- To find the conservatively large sample size needed, set $p = 0.5$.
- Confidence interval for a population mean (when σ unknown):

$$\overline{x} \pm \frac{t^* s}{\sqrt{n}} \qquad df = n - 1$$

- z-test statistic for hypothesis testing of a population proportion:

$$z^* = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- t-test statistic for hypothesis testing of a population mean (when σ unknown):

$$t^* = \frac{\overline{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} \qquad df = n - 1$$

- Hypothesis testing for the difference of two means (Welch's T Test):

$$t^* = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$df = \text{smaller of } n_1 - 1 \text{ and } n_2 - 1$$

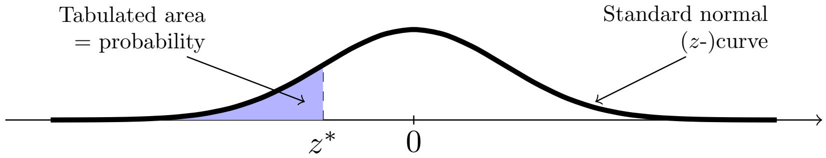
- Hypothesis testing for the difference of two proportions (2-Sample Z Test for Equal Proportions):

$$z^* = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}}$$

- Combined/pooled proportion:

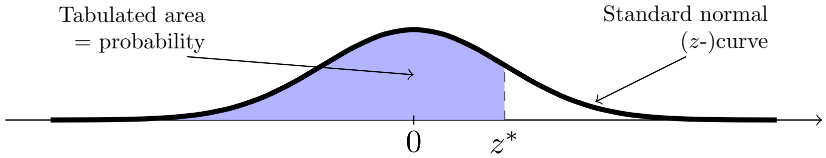
$$\hat{p}_c = \frac{n_1 \cdot \hat{p}_1 + n_2 \cdot \hat{p}_2}{n_1 + n_2} \qquad \text{or} \qquad \hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

Standard Normal Table



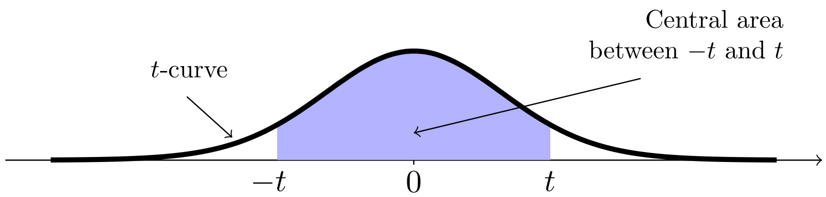
z^*	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
-3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Table



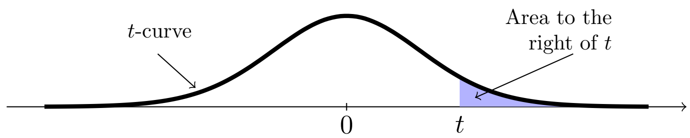
z^*	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

t-Distribution Table of Critical Values



degree of freedom	80%	90%	95%	96%	97%	98%	99%	99.8%	99.9%
	Central Area Captured / Confidence Level								
1	3.078	6.314	12.706	15.895	21.205	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	4.849	5.643	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	3.482	3.896	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	2.999	3.298	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	2.757	3.003	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	2.612	2.829	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.517	2.715	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.449	2.634	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.398	2.574	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.359	2.527	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.328	2.491	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.303	2.461	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.282	2.436	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.264	2.415	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.249	2.397	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.235	2.382	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.224	2.368	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.214	2.356	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.205	2.346	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.197	2.336	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.189	2.328	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.183	2.320	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.177	2.313	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.172	2.307	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.167	2.301	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.162	2.296	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.158	2.291	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.154	2.286	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.150	2.282	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.147	2.278	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.123	2.250	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.109	2.234	2.403	2.678	3.261	3.496
60	1.296	1.671	2.000	2.099	2.223	2.390	2.660	3.232	3.460
70	1.294	1.667	1.994	2.093	2.215	2.381	2.648	3.211	3.435
80	1.292	1.664	1.990	2.088	2.209	2.374	2.639	3.195	3.416
90	1.291	1.662	1.987	2.084	2.205	2.368	2.632	3.183	3.402
100	1.290	1.660	1.984	2.081	2.201	2.364	2.626	3.174	3.390
110	1.289	1.659	1.982	2.078	2.199	2.361	2.621	3.166	3.381
120	1.289	1.658	1.980	2.076	2.196	2.358	2.617	3.160	3.373
z-critical $= \infty$	1.282	1.645	1.960	2.054	2.170	2.326	2.576	3.090	3.291

Tail Areas for t Curves



t^* \ df	**1**	**2**	**...**	**29**	**30**	**40**	**50**	**...**	**110**	**120**
0.0	0.500	0.500	...	0.500	0.500	0.500	0.500	...	0.500	0.500
0.1	0.468	0.465	...	0.461	0.461	0.460	0.460	...	0.460	0.460
0.2	0.437	0.430	...	0.421	0.421	0.421	0.421	...	0.421	0.421
0.3	0.407	0.396	...	0.383	0.383	0.383	0.383	...	0.382	0.382
0.4	0.379	0.364	...	0.346	0.346	0.346	0.345	...	0.345	0.345
0.5	0.352	0.333	...	0.310	0.310	0.310	0.310	...	0.309	0.309
0.6	0.328	0.305	...	0.277	0.277	0.276	0.276	...	0.275	0.275
0.7	0.306	0.278	...	0.245	0.245	0.244	0.244	...	0.243	0.243
0.8	0.285	0.254	...	0.215	0.215	0.214	0.214	...	0.213	0.213
0.9	0.267	0.232	...	0.188	0.188	0.187	0.186	...	0.185	0.185
1.0	0.250	0.211	...	0.163	0.163	0.162	0.161	...	0.160	0.160
1.1	0.235	0.193	...	0.140	0.140	0.139	0.138	...	0.137	0.137
1.2	0.221	0.177	...	0.120	0.120	0.119	0.118	...	0.116	0.116
1.3	0.209	0.162	...	0.102	0.102	0.101	0.100	...	0.098	0.098
1.4	0.197	0.148	...	0.086	0.086	0.085	0.084	...	0.082	0.082
1.5	0.187	0.136	...	0.072	0.072	0.071	0.070	...	0.068	0.068
1.6	0.178	0.125	...	0.060	0.060	0.059	0.058	...	0.056	0.056
1.7	0.169	0.116	...	0.050	0.050	0.048	0.048	...	0.046	0.046
1.8	0.161	0.107	...	0.041	0.041	0.040	0.039	...	0.037	0.037
1.9	0.154	0.099	...	0.034	0.034	0.032	0.032	...	0.030	0.030
2.0	0.148	0.092	...	0.027	0.027	0.026	0.025	...	0.024	0.024
2.1	0.141	0.085	...	0.022	0.022	0.021	0.020	...	0.019	0.019
2.2	0.136	0.079	...	0.018	0.018	0.017	0.016	...	0.015	0.015
2.3	0.131	0.074	...	0.014	0.014	0.013	0.013	...	0.012	0.012
2.4	0.126	0.069	...	0.012	0.011	0.011	0.010	...	0.009	0.009
2.5	0.121	0.065	...	0.009	0.009	0.008	0.008	...	0.007	0.007
2.6	0.117	0.061	...	0.007	0.007	0.006	0.006	...	0.005	0.005
2.7	0.113	0.057	...	0.006	0.006	0.005	0.005	...	0.004	0.004
2.8	0.109	0.054	...	0.004	0.004	0.004	0.004	...	0.003	0.003
2.9	0.106	0.051	...	0.004	0.003	0.003	0.003	...	0.002	0.002
3.0	0.102	0.048	...	0.003	0.003	0.002	0.002	...	0.002	0.002
3.1	0.099	0.045	...	0.002	0.002	0.002	0.002	...	0.001	0.001
3.2	0.096	0.043	...	0.002	0.002	0.001	0.001	...	0.001	0.001
3.3	0.094	0.040	...	0.001	0.001	0.001	0.001	...	0.001	0.001
3.4	0.091	0.038	...	0.001	0.001	0.001	0.001	...	0.000	0.000
3.5	0.089	0.036	...	0.001	0.001	0.001	0.000	...	0.000	0.000
3.6	0.086	0.035	...	0.001	0.001	0.000	0.000	...	0.000	0.000
3.7	0.084	0.033	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.8	0.082	0.031	...	0.000	0.000	0.000	0.000	...	0.000	0.000
3.9	0.080	0.030	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.0	0.078	0.029	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.1	0.076	0.027	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.2	0.074	0.026	...	0.000	0.000	0.000	0.000	...	0.000	0.000
4.3	0.073	0.025	...	0.000	0.000	0.000	0.000	...	0.000	0.000