Practice Problems for Math Success

Graphs of Sine and Cosine Functions

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

1. In each of the following parts, circle the **ONE** alternative that best completes the sentence.

   (a) The function which has the same minimum value as $f(x) = 2\sin(3x) + 5$ is
   - i. $g(x) = 2\sin(3x + 5)$.
   - ii. $h(x) = 2 + 5\sin(3x)$.
   - iii. $j(x) = 5 + 2\sin(2x + 1)$.
   - iv. $k(x) = 7\sin(2x - 1)$.
   - v. None of these.

   (b) The function which has the same maximum value as $f(x) = 2\sin(3x) + 5$ is
   - i. $g(x) = 2\cos(3x + 5)$.
   - ii. $h(x) = 2 - 5\cos(2x)$.
   - iii. $j(x) = 7 + 2\sin(2x + 1)$.
   - iv. $k(x) = 7\sin(3x) + 1$.
   - v. None of these.

   (c) The period of $g(x) = 2\sin(3x + 6) + 5$ is
   - i. 3.
   - ii. $2\pi/3$.
   - iii. 6π.
   - iv. $1/3$.
   - v. None of the above.

   (d) We obtain the graph of $g(x) = 2\sin(3x + 6) + 5$ from the graph of $f(x) = 2\sin(3x)$ by shifting $y = f(x)$
   - i. to the right by 2 units and up by 5 units.
   - ii. to the left by 2 units and up by 5 units.
   - iii. to the right by 6 units and up by 5 units.
   - iv. to the left by 6 units and up by 5 units.
   - v. None of the above.

2. The average monthly temperature in Washington, D.C. can be approximated by the following function

$$W(t) = 57 + 23\sin\left(\frac{\pi}{6}t - \frac{1}{2}\right),$$

and the average monthly temperature in Baltimore, Maryland can be approximated by the following function

$$B(t) = 55 + 25\sin\left(\frac{\pi}{6}t - \frac{1}{2}\right),$$

where t is measured in months.

(a) Fill in the blanks. You do not need to show work.

i. The period of $W(t)$ is _____.

ii. The amplitude of $W(t)$ is _____.

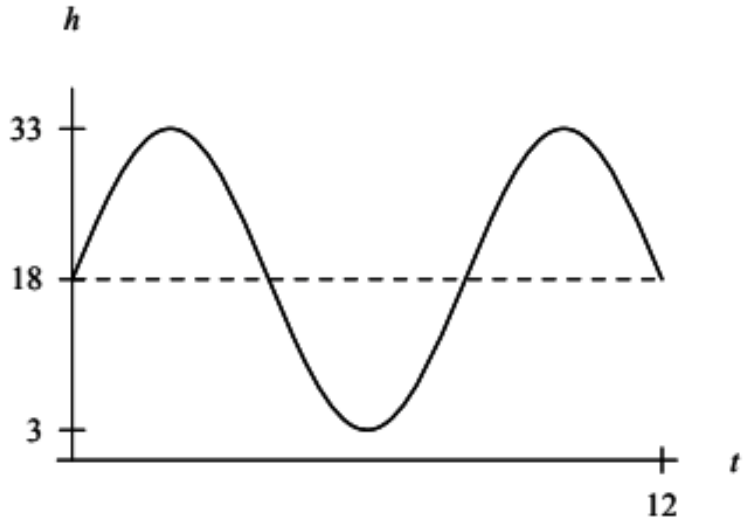
iii. The midline of $W(t)$ is _____.

(b) Decide whether each of the following statements is TRUE or FALSE. Explain your reasoning.

i. The minimum temperature in Washington is higher than the minimum temperature in Baltimore.

ii. The maximum temperature in Washington is higher than the maximum temperature in Baltimore.

3. The graph below describes your height, $h = f(t)$, above the ground on a ferris wheel (that turns in a counter-clockwise direction), where h is in meters and t is time in minutes.



Fill in the blanks.

(a) How high above the ground are you at time $t = 0$? _____

(b) What is your position on the wheel at $t = 0$? (That is, what o'clock?)

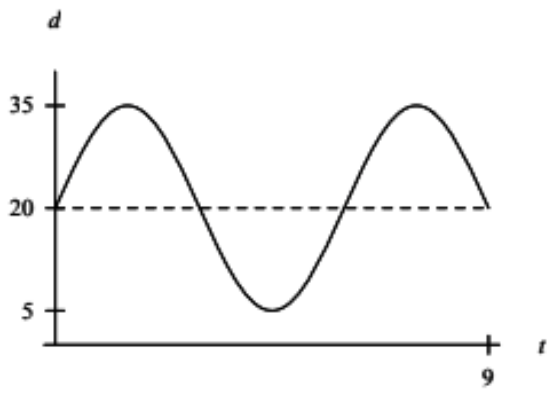
_____

(c) What is the radius of the wheel? _____

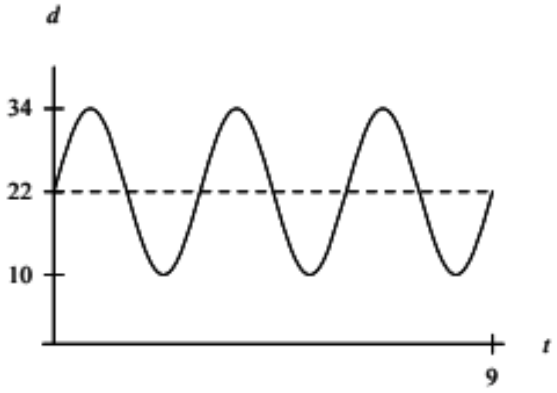
(d) How long does one revolution take? _____

4. Consider a weight suspended from the ceiling by a spring. Let d be the distance in centimeters **from the ceiling** to the center of the mass. When the weight is disturbed and begins to oscillate, then d is a periodic function of t, time in seconds, so $d = f(t)$.

   The figures below describe the motion of two different weights, A and B, attached to two different springs. Based on these graphs, fill in the blanks. You do not have to give any explanations.



(a) Weight A



(b) Weight B

   (a) Which weight, when not in motion, is closer to the ceiling? _____

   (b) Which weight gets closer to the ceiling during the oscillations? _____

   How close does it get? _____

   (c) Which weight makes larger oscillations? _____

   (d) Which weight makes faster oscillations? _____

5. A ferris wheel is 30 meters in diameter and must be boarded from a platform that is 4 meters above the ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes one full revolution every 3 minutes. At $t = 0$ you are in the 3 o'clock position and ascending.

   Find a formula, using a **sine function**, for your height above ground after t minutes on the ferris wheel. Show all your work.