Practice Assessment

Derivatives and the Shape of a Graph

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

Test for Concavity: Let f be a function that is twice differentiable over an interval I.

i. If $f''(x) > 0$ for all $x \in I$, then f is **concave up** over I.

ii. If $f''(x) < 0$ for all $x \in I$, then f is **concave down** over I.

Inflection Point: If f is continuous at x_0 and f changes concavity at x_0, then the point $(x_0, f(x_0))$ is an **inflection point** of f.

1. Determine the intervals where the following functions are concave up and concave down. Also find all points of inflection.

   (a) $f(x) = x^4 - 2x^2 + 3$

   (b) $f(x) = x^{\frac{1}{3}}(x+4)$

   (c) $f(x) = 5x^{2/3} - 2x^{5/3}$

(d) $f(x) = e^{4-x^2}$

(e) $f(x) = xe^{2x}$

(f) $f(x) = \frac{4}{3}x - \tan(x) \quad \text{on} \quad [0, 2\pi]$

2. Sketch a complete graph of each function. Label any critical points as absolute or local extrema as ordered pairs. Label any points of inflection as ordered pairs. Label any asymptotes with the correct equation(s).

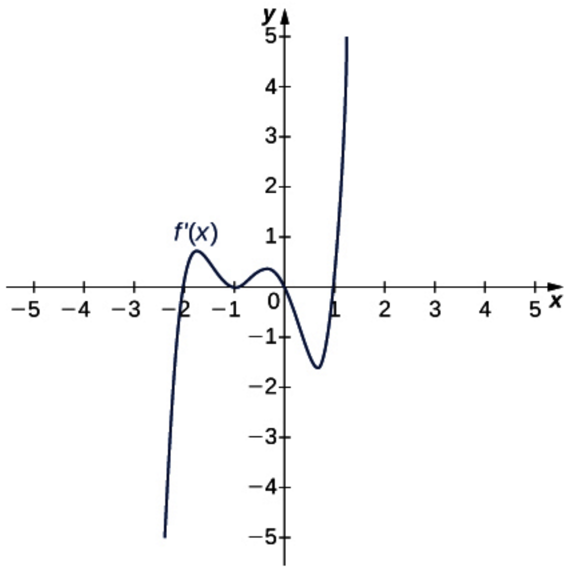
   (a) $f(x) = 2x + \dfrac{2}{x^2}$

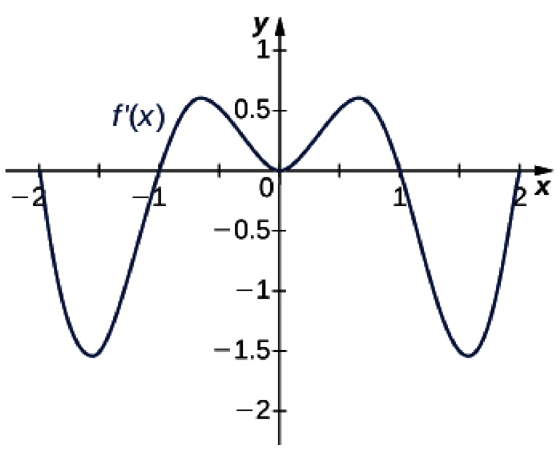
   (b) $f(x) = x(x-4)^3$

(c) $f(x) = x^4 + \frac{4}{3}x^3 - 4x^2$

(d) $f(x) = \frac{x^2}{x^2 - 4}$

3. Analyze each of the graph of f' below, then list all intervals where f is increasing or decreasing,





4. Analyze each of the graph of f' below, then list all:
   (a) Intervals where f is concave up or concave down,
   (b) Inflections points.