Practice Assessment
Maximum and Minimum Values

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

Critical Points and Critical Numbers: We say that c is a **critical number** of f if $f'(c) = 0$ or $f'(c)$ is undefined. We call the point $(c, f(c))$ a **critical point**.

Note that these two terms are often used interchangeably.

1. For each of the following functions, find all critical points.

   (a) $s(t) = t^3(t-5)^2$

   (b) $h(z) = \sqrt{3 + 2z - z^2}$

   (c) $y = \arccos(t^2)$

   (d) $g(x) = x^2 \ln(x)$

Locating Absolute Extrema over a Closed Interval: Consider a continuous function f defined over the closed interval $[a, b]$.

(a) Evaluate f at the endpoints $x = a$ and $x = b$.

(b) Find all critical points of f that lie over the interval (a, b) and evaluate f at those critical points.

(c) Compare all values found in (a) and (b). The largest of these values is the **absolute maximum** of f. The smallest of these values is the **absolute minimum** of f.

2. For each of the following functions, find the absolute extrema over the indicated closed interval.

(a) $f(x) = \sin\left(\frac{x}{2}\right), \quad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

(b) $r(x) = x^{1/3}(x + 4), \quad [-8, 1]$

(c) $h(x) = \ln(x + 1), \quad [0, 3]$

(d) $g(x) = x^{16/3} - 5x^{10/3}, \quad [-2, 2]$