Practice Assessment The Mean Value Theorem

These practice problems are designed to help you prepare for our course exams and assess your understanding of the course material at the expected level. Aim to complete them in class, during tutoring, office hours, or on your own, and try to solve them without notes or a calculator, just like on the actual exams. Remember, practice makes perfect, so don't hesitate to ask for help if you get stuck.

The Mean Value Theorem: Let f be continuous over the closed interval [a, b] and differentiable over the open interval (a, b). Then, there exists at least one point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

1. For each of the following functions, find all numbers c that satisfy the conclusion of the Mean Value Theorem on the indicated interval.

(a)
$$p(x) = x^2 + 2x - 3$$
, $-3 \le x \le 0$

(b)
$$r(x) = \frac{1}{(x-2)^2}$$
, $3 \le x \le 5$

(c)
$$h(x) = 3\sin(2x), \quad 0 \le x \le \frac{\pi}{2}$$

(d)
$$h(x) = \ln(x^2 + 1), -2 \le x \le \frac{1}{2}$$