Practice Assessment

Linear Approximations and Differentials

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

Linear Approximation: The linear approximation, tangent line approximation, or linearization of a function f near a point $x = a$ is given by

$$f(x) \approx L(x) = f(a) + f'(a)(x - a).$$

In other words, the linear function $L(x)$, which is the equation of the tangent line of f at $x = a$, is used to approximate the function $f(x)$ for values near $x = a$.

1. Find the linear approximation L of the following functions at the indicated points.

   (a) $r(x) = \dfrac{x-1}{(x+1)^2}, \quad x = 1$

   (b) $h(x) = 3\sin(2x), \quad x = \dfrac{\pi}{4}$

   (c) $h(x) = \ln(x^2), \quad x = e$

2. Find an approximation to $f(9)$ if $f(10) = 6$ and $f'(10) = -2$.

3. Find an approximation to $f(36)$ if $f(39) = 7$ and $f'(39) = 5$.

4. Approximate the following quantities using a linearization.

   (a) $\sqrt{8.9}$

   (b) $\ln(1.1)$

Differential: The differential form of $y = f(x)$ is $dy = f'(x)dx$.

5. Find the differential of each of the following functions.

   (a) $y = \sec(1 - x^2)$

   (b) $y = \dfrac{\sqrt{x}}{x^{2/3} + x}$

6. Let $y = (x^2 + x)^2$. Find the differential dy when $x = 1$ and $dx = 0.2$.