Practice Assessment
L'Hopitals Rule

These **practice problems** are designed to help you **prepare for our course exams** and **assess your understanding** of the course material at the expected level. Aim to complete them **in class, during tutoring, office hours, or on your own**, and try to solve them **without notes or a calculator**, just like on the **actual exams**. Remember, **practice makes perfect**, so don't hesitate to **ask for help** if you get stuck.

L'Hôpital's Rule: Let f and g be differentiable functions where defined, except possibly at $x = a$. If

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\infty}{\infty},$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

This result also hold if we are considering one-sided limits, or if $a = -\infty$ or $a = \infty$.

1. For each of the following limits determine which indeterminate form the expression corresponds to, then calculate the limit using L'Hôpital's Rule.

   (a) $\lim_{x\to 1} \frac{1-x}{e^x - e}$

   (b) $\lim_{x\to\infty} \frac{\sqrt{x}-\sqrt{5}}{\sqrt{x}+\sqrt{5}}$

   (c) $\lim_{x\to 0} \frac{9x^3}{xe^{\pi x}}$

   (d) $\lim_{x\to 0^+} \frac{1-\cos\sqrt{x}}{\sin(x)}$

(e) $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)$

(f) $\displaystyle\lim_{x\to 1^+}\left(\frac{1}{1-x}-\frac{x}{\sqrt{x-1}}\right)$

(g) $\displaystyle\lim_{x\to 0} x\cot(x)$

(h) $\displaystyle\lim_{x\to\infty}\left(\ln\sqrt{4x+2}-\ln\sqrt{x+3}\right)$

(i) $\lim_{x \to 0^+} \frac{\ln(2x)}{2x}$

(j) $\lim_{x \to \frac{\pi}{4}} \frac{\sin(x) - \cos(x)}{\tan(x) - 1}$

(k) $\lim_{x \to \frac{\pi}{2}^+} (\sec(x) - \tan(x))$