UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

AMAT112 Calculus Exam 1A Fall 2024

Print Name:

UAlbany Email:

Instructor's Name:

Directions: You have **75 minutes** to answer the following questions. ***You must show all necessary work*** as neatly and clearly as possible and clearly indicate your final answers.

No calculators, notes, textbooks, mobile phones or other aids are allowed. Do not detach pages.

Problem	Possible	Points
1	6	
2	8	
3	6	
4	6	
5	7	
6	6	
7	8	
8	8	
Total	55	

YOUR SIGNATURE:

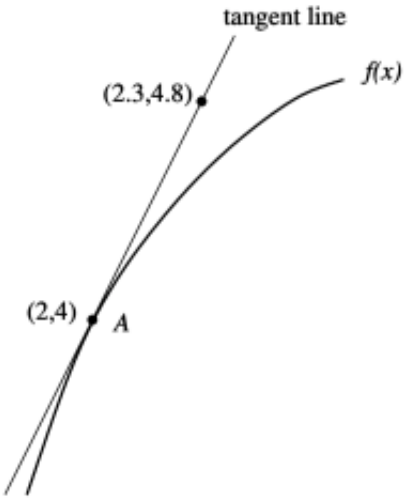
(1) The total cost, in dollars, of repaying a student loan at an interest rate of $r\%$ per year is $C = f(r)$.

(a) (3 Points) What are the units of $f'(r)$?

(b) (3 Points) In the context of the problem, what is the meaning of $f'(7) = 1200$?

YOUR SIGNATURE:

(2) Use the figure below to fill in the blanks about the function f at the point A.



(a) (2 Points) $f($_____$) =$ _____.

(b) (3 Points) $f'($_____$) =$ _____.

(c) (3 Points) The equation of the tangent line of f at the point A is _____.

YOUR SIGNATURE:

(3) Evaluate the following limits. If the limit does not exists, write "DNE". (2 Points Each)

(a) $\lim_{x \to 2^+} \dfrac{x^2 - x + 6}{x - 4} =$

(b) $\lim_{x \to 8} \dfrac{x^2 - 10x + 16}{x - 8} =$

(c) $\lim_{x \to 25} \dfrac{25 - x}{5 - \sqrt{x}} =$

YOUR SIGNATURE:

(4) Evaluate the following limits. If the limit does not exists, write "DNE". (2 Points Each)

(a) $\lim_{s\to-\infty} \dfrac{9s^4 - 3s^2 - 7}{15s^4 + 17s + 1} =$

(b) $\lim_{x\to\infty} \dfrac{e^{-x} - 2}{e^{-x} - 5} =$

(c) $\lim_{t\to-\infty} \dfrac{\sqrt{t^2 - 1}}{2t - 2} =$

YOUR SIGNATURE:

(5) (7 Points) Let

$$f(x) = \frac{1}{x+5}.$$

Find the $f'(x)$ **algebraically** using the limit definition of derivative. Show all your work. No credit given for applying the rules of differentiation.

YOUR SIGNATURE:

(6) Let

$$h(x) = \begin{cases} x^2 - 9 & x < c \\ 4x - 13 & x \geq c \end{cases}.$$

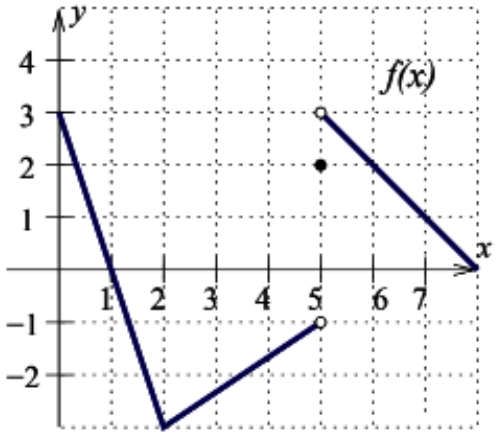
where c is some constant.

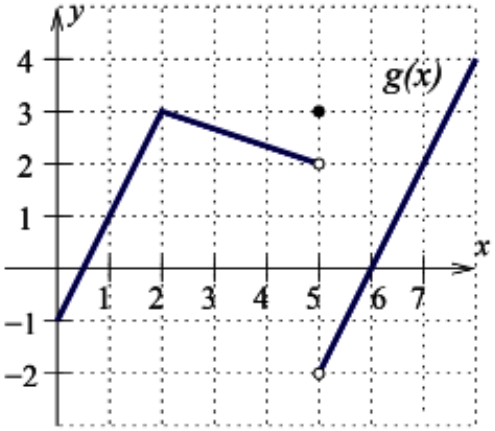
(a) (2 Points) $\lim_{x \to c^-} h(x) =$ _____

(b) (2 Points) $\lim_{x \to c^+} h(x) =$ _____

(c) (2 Points) For what value(s) of c is $h(x)$ continuous?

YOUR SIGNATURE:





(7) (2 Points Each) Using the graphs of f and g above, evaluate each limit, or write DNE if it does not exists.

(a) $\lim_{x \to 5^-} [f(x) - 5] =$ _____

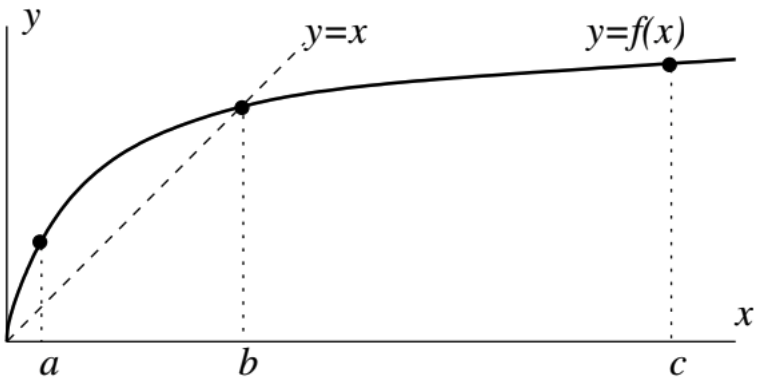
(b) $\lim_{x \to 5^+} \dfrac{f(5)}{g(x)} =$ _____

(c) $\lim_{x \to 5^-} [f(x) + g(x)] =$ _____

(d) $\lim_{x \to 5} [f(x) + g(x)] =$ _____

YOUR SIGNATURE:

(8) The graph of $f(x)$ is given in the figure below.



(2 Points Each) For each of the following pairs of numbers, circle the larger one. You do not need to justify your answer.

(a) $f'(a)$ OR The slope of the tangent line at $x = b$.

(b) $f'(b)$ OR $\dfrac{f(b) - f(a)}{b - a}$

(c) $f'(c)$ OR The number 0.

(d) $\dfrac{f(b) - f(a)}{b - a}$ OR The number 1.

YOUR SIGNATURE:

Formulas you might find useful

- **The derivative of a function at a point $x = a$ is**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{OR} \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- **The equation of the tangent line** of a function f at a is given by

$$y = f(a) + f'(a)(x - a)$$