AMAT 100
Worksheet #3

1 Consider the quadratic function:

$$Q(x) = 7x^2 - 28x + 23$$

$a = 7,\ b = -28,\ c = 23$

(a) Rewrite $Q(x)$ in the form $a(x-h)^2 + k$

$$h = \frac{-b}{2a} = \frac{-(-28)}{2(7)} = \frac{28}{14} = \boxed{2}$$

$$\begin{aligned} k = Q(h) = Q(2) = 7(2)^2 - 28(2) + 23 &= 7(4) - 28(2) + 23 \\ &= 28 - 28(2) + 23 \\ &= 28(1-2) + 23 \\ &= 28(-1) + 23 \\ &= -28 + 23 \\ &= \boxed{-5} \end{aligned}$$

$$\boxed{Q(x) = \underset{a}{7}(x - \underset{h}{2})^2 \underset{k}{-5}}$$

(b) What is the maximum value of $Q(x)$?

Since $a > 0$, and $\mathrm{dom}(Q(x)) = (-\infty, \infty)$, $Q(x)$ has no maximum value.

(c) What is the y-intercept of the graph of $Q(x)$?

$$Q(0) = 7(0-2)^2 - 5$$
$$= 7(-2)^2 - 5$$
$$= 7(4) - 5$$
$$= 28 - 5$$
$$= \boxed{23}$$

(d) What are the zeroes of $Q(x)$?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-28) \pm \sqrt{(28)^2 - 4(7)(23)}}{2(7)}$$

$$= \frac{28 \pm \sqrt{(28)^2 - 28(23)}}{14}$$

$$= \frac{28 \pm \sqrt{(28)(28) - 28(23)}}{14}$$

$$= \frac{28 \pm \sqrt{28(28-23)}}{14}$$

$$= \frac{28 \pm \sqrt{28(5)}}{14}$$

1(d) cont.

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$$= \frac{28 \pm \sqrt{4 \cdot 7 \cdot 5}}{14}$$

$$= \frac{28 \pm 2\sqrt{7 \cdot 5}}{14}$$

$$= 2 \pm \frac{\sqrt{35}}{7}$$

They are $\boxed{x = 2 + \frac{\sqrt{35}}{7}}$ and $\boxed{x = 2 - \frac{\sqrt{35}}{7}}$

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(a) Identify the zeroes and its multiplicities for the function:

$$p(x) = (x-545)^5 (x+46)^4 (x-\pi)^3 (x-\tfrac{\pi}{4}) (x+\tfrac{1}{2})^{2024}$$

The zeroes are:

$x = 545$; there are five of them.
$x = -46$; there are four of them.
$x = \pi$; there are three of them.
$x = \frac{\pi}{4}$; there is one.
$x = \frac{-1}{2}$; there are 2024 of them.

(b)

(i) $\boxed{\text{No.}}$ $L(x) = 7 \cdot \left(\frac{1}{2}\right)^x$ is not a power function.

(ii) $P(x) = x^{4n}\sqrt{\frac{\pi}{7x^{2n}}}$

$$= \sqrt{\frac{x^{8n}\pi}{7x^{2n}}}$$

$$= \sqrt{\frac{\pi}{7}x^{6n}}$$

$$= \frac{\sqrt{\pi}}{\sqrt{7}}x^{3n} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \frac{\sqrt{7\pi}}{7}x^{3n}$$

$\boxed{\text{Yes.}}$ $P(x)$ is a power function. $\boxed{k = \frac{\sqrt{7\pi}}{7}, \ p = 3n}$

(3) Use long division to find the quotient and remainder.

$$(15x^5+4x^4+2x^2+7) \div (x-10)$$

$$\begin{array}{r l}
 & 15x^4 + 154x^3 + 1540x^2 + 15402x + 154020 \\
x-10 \,\big) & 15x^5 + 4x^4 + 0x^3 + 2x^2 + 0x + 7 \\
 & -(15x^5 - 150x^4) \\
\hline
 & 154x^4 + 0x^3 + 2x^2 + 0x + 7 \\
 & -(154x^4 - 1540x^3) \\
\hline
 & +1540x^3 + 2x^2 + 0x + 7 \\
 & -(1540x^3 - 15400x^2) \\
\hline
 & 15402x^2 + 0x + 7 \\
 & -(15402x^2 - 154020x) \\
\hline
 & 154020x + 7 \\
 & -(154020x - 1540200) \\
\hline
 & \boxed{1540207}
\end{array}$$

Quotient: $15x^4+154x^3+1540x^2+15402x+154020$

Remainder: 1540207

(4) Consider the rational function:

$$k(x) = \frac{x+1}{x(x-2)(x-3)(x+5)}$$

(a) Find all x and y-intercepts of $k(x)$.

$$k(0) = \frac{0+1}{0(0-2)(0-3)(0+5)} = \frac{1}{0} \Rightarrow \boxed{k(x) \text{ has no } y\text{-intercept.}}$$

$$0 = \frac{x+1}{x(x-2)(x-3)(x+5)} \Rightarrow x+1=0 \Rightarrow \boxed{x=-1 \text{ is a } x\text{-intercept}}$$

(b) Identify all vertical asymptotes.

x, $(x-2)$, $(x-3)$, and $(x+5)$ are in the demominator only.

$\Rightarrow$ $x=0$, $x=2$, $x=3$, $x=-5$ are all vertical asymptotes of $k(x)$

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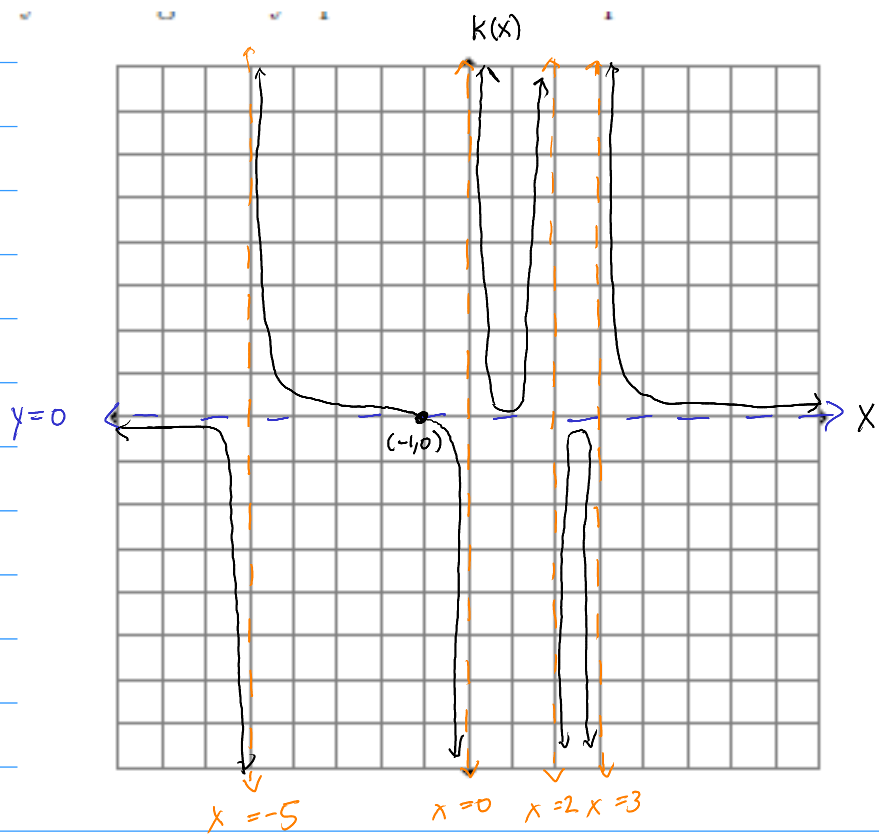
(c) Identify all horizontal asymptotes of $k(x)$

This is a Case I. The degree of the numerator < the degree of the denominator. $\Rightarrow$ $y=0$ is a horizontal asymptote.

(d) Sketch $k(x)$ clearly labeling all asymptotes and intercepts.



	$(-\infty,-5)$	$(-5,-1)$	$(-1,0)$	$(0,2)$	$(2,3)$	$(3,\infty)$
$(x+1)$	−	−	+	+	+	+
x	−	−	−	+	+	+
$(x-2)$	−	−	−	−	+	+
$(x-3)$	−	−	−	−	−	+
$(x+5)$	−	+	+	+	+	+
$x(x-2)(x-3)(x+5)$	+	−	−	+	−	+
$f(x)$	−	+	−	+	−	+

4d Table interpreted.

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	$(-\infty,-5)$	$(-5,-1)$	$(-1,0)$	$(0,2)$	$(2,3)$	$(3,\infty)$
$(x+1)$	$-$	$-$	$+$	$+$	$+$	$+$
x	$-$	$-$	$-$	$+$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	$-$	$+$	$+$
$(x-3)$	$-$	$-$	$-$	$-$	$-$	$+$
$(x+5)$	$-$	$+$	$+$	$+$	$+$	$+$
$x(x-2)(x-3)(x+5)$	$+$	$-$	$-$	$+$	$-$	$+$
$f(x)$	$-$	$+$	$-$	$+$	$-$	$+$

$\Rightarrow$ as $x \to -5^-$, $f(x) \to -\infty$

as $x \to -5^+$, $f(x) \to +\infty$

as $x \to -0^-$, $f(x) \to -\infty$

as $x \to -0^+$, $f(x) \to +\infty$

as $x \to 2^-$, $f(x) \to +\infty$

as $x \to 2^+$, $f(x) \to -\infty$

as $x \to 3^-$, $f(x) \to -\infty$

as $x \to 3^+$, $f(x) \to +\infty$

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⑤ Consider the function $a(x) = \sqrt{14x+20}$

(a) What is the domain of $a(x)$?

$$14x + 20 \geq 0$$

$$14x \geq -20$$

$$x \geq -20/14$$

$$x \geq -10/7$$

$$\boxed{[-10/7, \infty)}$$

(b) Find the inverse of $a(x)$

$$x = \sqrt{14y + 20}$$

$$\Rightarrow x^2 = 14y + 20$$

$$\Rightarrow x^2 - 20 = 14y$$

$$\Rightarrow y = \frac{x^2 - 20}{14}$$

$$\boxed{a^{-1}(x) = \frac{x^2-20}{14}, \ x \geq 0}$$

5 cont.

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(c) What is the domain of $a^{-1}(x)$?

The domain of $a^{-1}(x)$ is the range of $a(x)$, which is:

$\boxed{[0,\infty)}$

(d) What is the range of $a(x)$?

The range of $a(x)$ is the domain of $a^{-1}(x)$ which is:

$\boxed{[0,\infty)}$