Grade: /10

AMAT113: Calculus II
Worksheet 4

Due: Wednesday, March 13, in Class or Digitally

Name:

UAlbany Email:

—— Instructions ——

- This homework should be submitted in class or digitally on the date listed above.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as Microsoft Lens to take pictures of your work with your phone and convert them into a single pdf file.
- **You must show all work**. You may receive zero or reduced points for insufficient work. **Your work must be neatly organized and written**. You may receive zero or reduced points for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- **Put a box or circle around your final answer** for each question.
- The problems on this assignment will be **graded on correctness and completeness**.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.

1. The substitution rule (or "u-sub") and integration-by-parts are two of the most indispensable tools when calculating integrals, but they might not always be necessary or, possibly, could both come up in the solution of an integral. Consider $\int e^{\sqrt{x}} dx$.

   (a) If we let $u = e^{\sqrt{x}}$ then it follows that $\ln u = \sqrt{x}$. Use that last equation to express dx **only** in terms of u and du.

   (b) Using the form of dx obtained above set up the integral resulting from that substitution. Evaluate it and rewrite in terms of x.

(c) If, on the other hand, we let $u = \sqrt{x}$ then it follows that $dx = 2udu$. Evaluate $\int 2ue^u du$ using integration-by-parts and then rewrite in terms of x.

2. In the following a is a non-zero constant. Try using the preliminary trick of completing the square followed by various substitution strategies to evaluate

$$\int \frac{dx}{\sqrt{x^2 + 2ax}}$$

3. Use polynomial long division and the method of partial fraction decomposition to compute

$$\int_2^3 \frac{x^6 - x^5 - x^4 + 3x^3 - 2x^2 + x + 1}{x(x+1)(x-1)^2} dx$$

4. There is a paradoxical surface of revolution known as **Gabriel's Horn** and, as it happens, we can explore this paradox with improper integrals!

   (a) Sketch a portion of the surface of revolution obtained by revolving the graph of $y = \frac{1}{x}$, defined on the interval $[1, \infty)$, about the x-axis.

   (b) Using the method of *washers* calculate the volume of the region inside the horn.

(c) Set up, **but do not evaluate**, the improper integral needed to calculate the *surface area* of the horn.

(d) The improper integral $\int_1^\infty \frac{dx}{x}$ is well-known to be divergent. Using your previous answer(s) and the Comparison Theorem for improper integrals, which of the following assessments accurately describe Gabriel's Horn? (SA is "surface area")

(i) finite SA and volume

(ii) finite SA enclosing an infinite volume

(iii) infinite SA enclosing a finite volume

(iv) infinite SA and volume