AMAT100: Precalculus
Worksheet 4

SOLUTIONS

Due: Due Friday 3/29 digitally

Out of 10 points

—— Instructions ——

- This homework should be submitted in class or digitally on the date listed above.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as Microsoft Lens to take pictures of your work with your phone and convert them into a single pdf file.
- **You must show all work**. You may receive zero or reduced points for insufficient work. **Your work must be neatly organised and written**. You may receive zero or reduced points for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- **Put a box or circle around your final answer** for each question.
- The problems on this assignment will be **graded on correctness and completeness**.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.

1. Some values of the function $f(x)$ are given in the table below.

x	$f(x)$
-1	-54
2	k
3	$-\frac{2}{3}$

Find the value of k if

(a) $f(x)$ is linear.

$f(x) = mx + b$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\frac{2}{3} - (-54)}{3-(-1)} = \frac{53\frac{1}{3}}{4} = \frac{53}{4} + \frac{1}{12} = \frac{159}{12} + \frac{1}{12} = \boxed{\frac{160}{12}}$ (or $\frac{40}{3}$)

+½ pt for the correct m; or +¼ pt for work w/ formula & errors in arithmetic

$f(3) = \left(\frac{40}{3}\right)(3) + b \Rightarrow f(3) = (40) + b$

$\Rightarrow -\frac{2}{3} = (40) + b \Rightarrow b = \frac{-2}{3} - \frac{120}{3} = \boxed{\frac{-122}{3}}$

+½ pt for the correct b; or +¼ pt for work in the right direction w/ errors in arithmetic not from calculating m.

$f(x) = \left(\frac{40}{3}\right)x - \left(\frac{122}{3}\right)$

$f(2) = \left(\frac{40}{3}\right)(2) + \left(\frac{-122}{3}\right) = \frac{80}{3} - \frac{122}{3} = \frac{-42}{3} = \boxed{-14} = k$

+⅛ pts for correct computation of k, based on the student's m & b

(b) $f(x)$ is exponential.

$f(x) = ab^x$

$\Rightarrow f(-1) = ab^{-1} \Rightarrow -54 = ab^{-1} \Rightarrow \boxed{a = -54b}$

$\Rightarrow f(3) = ab^3 \Rightarrow -\frac{2}{3} = ab^3 \Rightarrow -\frac{2}{3} = (-54b)b^3 \Rightarrow \frac{-2}{3} = -27 \cdot 2b^4$

$= (-3)^3 \cdot 2b^4$

+½ a point for some attempt at a system of equations.

$\Rightarrow b^4 = \frac{-2}{3 \cdot (-3)^3 \cdot 2} = \frac{-1}{-1 \cdot 34}$

$\Rightarrow \boxed{b = \frac{1}{3}}$ +¼ pts for correct b.

$f(x) = \left(\frac{-54}{3}\right)\left(\frac{1}{3}\right)^x = (-18)\left(\frac{1}{3}\right)^x$

$f(2) = (-18)\left(\frac{1}{3}\right)^2 = (-18)\left(\frac{1}{9}\right) = \boxed{-2} = k$

+⅛ pts for correct computation of k, based on student's a and b.

+¼ pts for correct a

2.25

2. Transformations of Exponential and Logarithmic Functions

(a) Let $f(x) = 7^x$.
Let $g(x)$ be the following transformation of $f(x)$:
Shift $f(x)$ down by 15 units and to the left by 34 units. Write the equation for $g(x)$. *Do not simplify.*

$g(x) = 7^{(x+34)} - 15$
(+1/4 pts) (+1/4 pts)

Note: +1/8 of a point if a right shift of 34 units was made instead.
+1/8 of a point if a upward shift of 15 units was made instead.

(b) Let $h(x)$ be the following transformation of $g(x)$ Stretch your $g(x)$ from part (a) vertically by a factor of 17. Write the equation for $h(x)$ *Do not simplify.*

$h(x) = 17(7^{(x+34)} - 15)$
(+1/4 pts) (+1/4 pts for the student's g(x))

Note: +1/4 pts only if the student stretched his or her g(x) horizontally by a factor of 17 instead.

(c) Let $k(x) = \log(x)$. Let $v(x)$ be the reflection of $k(x)$ over the y-axis. Write the equation for $v(x)$. *Do not simplify.*

Note: +1/8 pts if the student reflected k(x) over the x-axis instead.

$v(x) = \log(-x)$
(+1/4 pts)

1.25

3. Rules of Logarithms

(a) Write as a single logarithm:

$$6\ln(2x) + \ln(a) - 13\ln(v) + \ln(b) + \ln(x)$$

$= \ln((2x)^6) + \ln(a) - 13\ln(v) + \ln(b) + \ln(x)$

$= \ln((2x)^6) + \ln(a) - \ln(v^{13}) + \ln(b) + \ln(x)$

$= \ln((2x)^6 \cdot a \cdot b \cdot x) - \ln(v^{13})$

$= \ln(2^6 x^6 abx) - \ln(v^{13})$

$= \boxed{\ln\left(\frac{2^6 x^7 ab}{v^{13}}\right)}$

+1/4 pts for applying $\log_b(MN) = \log_b(M) + \log_b(N)$ correctly
+1/4 pts for applying $\log_b(M/N) = \log_b(M) - \log_b(N)$ correctly
+1/4 pts for applying $\log_b(M^n) = n\log_b(M)$ correctly
+1/4 pts for combining all like terms in final answer

(b) Expand the following logarithm:

$$\log\left(\frac{ab}{cd}\right)^r$$

$= \log\left(\frac{a^r b^r}{c^r d^r}\right) = \log(a^r b^r) - \log(c^r d^r) = \log(a^r) + \log(b^r) - [\log(c^r) + \log(d^r)]$ (1)

$= \log(a^r) + \log(b^r) - \log(c^r) - \log(d^r)$ (2)

Either final answer is OK.

$= r\log(a) + r\log(b) - r\log(c) - r\log(d)$ (3)

$= r(\log(a) + \log(b) - \log(c) - \log(d))$ (4)

OR

$= r\log\left(\frac{ab}{cd}\right) = r\log(ab) - r\log(cd) = r[\log(a) + \log(b)] - r[\log(c) - \log(d)]$ (1)

$= r\log(a) + r\log(b) - r\log(c) - r\log(d)$ (2)

$= r(\log(a) + \log(b) - \log(c) - \log(d))$ (3)

+1/4 pts for applying $\log_b(MN) = \log_b(M) + \log_b(N)$ correctly
+1/4 pts for applying $\log_b(M/N) = \log_b(M) - \log_b(N)$ correctly
+1/4 pts for applying $\log_b(M^n) = n\log_b(M)$ correctly
+1/4 pts for distributing the negative sign correctly (1) to (2)

4

4. Exponential Equations
Solve for x.

$$2^{(x+7)} \cdot 32^{(x+12)} = 4^{(x-8)} \cdot 64^{(x+14)}$$

$$2^{(x+7)} \cdot 2^{5(x+12)} = 2^{2(x-8)} \cdot 2^{6(x+14)}$$

$$\Rightarrow \quad 2^{(x+7)+5(x+12)} = 2^{2(x-8)+6(x+14)}$$

① +1/2 point for converting all exponents to the same base.

② +1/2 for applying $X^{a+b} = X^a \cdot X^b$

$$\Rightarrow \quad (x+7)+5(x+12) = 2(x-8)+6(x+14)$$

③ +1/2 point for setting the exponents in ② equal to each other.

$$\Rightarrow \quad x+7+5x+60 = 2x-16+6x+84$$

$$\Rightarrow \quad 6x+67 = 8x+68$$

$$\Rightarrow \quad -1 = 2x$$

$$\Rightarrow \quad \boxed{x = \frac{-1}{2}}$$

④ -1/4 of a point for each noncontinual math error; -1/2 point max

+1/2 point if no errors in this section

2

5. Logarithmic Equations
Solve for x.

$$\log(x^2 + 16x - 27) = \log(2x^2 + 17x - 57)$$

$\Rightarrow x^2+16x-27 = 2x^2+17x-57$ ① } +½ points for setting the terms in the logs equal to each other.

$\Rightarrow x^2+x-30 = 0$ ② } +½ pts for correct algebra in getting one side to be zero, +¼ pts otherwise

$\Rightarrow (x+6)(x-5)=0$ ③ } +½ pts for correct factoring of ②

$\Rightarrow x=-6$ or $x=5$ ④ } +½ pts for identifying the correct roots of ③

OR

+1 pt for correct implementation of quadractic formula; -¼ pts for minor errors.

Check :

$5^2+16(5)-27 = 25+80-27 = 78 > 0$

$2(5)^2+17(5)-57 = 2(25)+17(5)-57$
$= 50+85-57$
$= 85-7 = 78 > 0$

$(-6)^2+16(-6)-27 = 36-96-27 < 0$, reject $x=-6$ as a solution for x.

$2(-6)^2+17(-6)-57 = 2(36)+17(-6)-57$
$= 72-102-57 < 0$, reject $x=-6$ as a solution for x

Ans: $x=5$

+⅛ point for rejecting $x=-6$ as a solution
+⅛ point for justification for rejecting $x=-6$ as a solution
+⅛ point for checking that $x=5$ is a solution
+⅛ point for stating / showing that $x=5$ is the ONLY solution
OR with whatever numbers the student has if an error was made.

(3) & (4) Quadratic Formula

$$X = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1^2-4(1)(-30)}}{2(1)} = \frac{-1}{2} \pm \frac{\sqrt{1+120}}{2} = \frac{-1}{2} \pm \frac{\sqrt{121}}{2}$$
$$= \frac{-1}{2} \pm \frac{11}{2}$$
$$= \frac{-1}{2} \pm 5.5$$
$$X = 5 \text{ or } X = -6$$

2-5