

AMAT113: Calculus II
Worksheet 1

Due: Wednesday, January 31, in Class or digitally

Name:

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— Instructions —

- This homework should be submitted in class or digitally on the date listed above.
- There are three main ways you might want to write up your work.
 - Write on this pdf using a tablet
 - Print this worksheet and write in the space provided
 - Write your answers on paper, clearly numbering each question and part.
 - * If using either of the last two options, you can use an app such as Microsoft Lens to take pictures of your work with your phone and convert them into a single pdf file.
- **You must show all work.** You may receive zero or reduced points for insufficient work. **Your work must be neatly organized and written.** You may receive zero or reduced points for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- **Put a box or circle around your final answer** for each question.
- The problems on this assignment will be **graded on correctness and completeness**.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.

1. Without evaluating the integrals, which value is larger? Justify your answer.

$$\int_0^{\pi/4} \frac{\cos(x)}{e^x + e^{-x}} dx \quad \text{or} \quad \int_0^{\pi/4} \frac{\sin(x)}{e^x + e^{-x}} dx$$

Since $e^x + e^{-x} > 0$ for all $x \in \mathbb{R}$
and $\cos(x) \geq \sin(x)$ for all $x \in [0, \pi/4]$,

$$\frac{\cos(x)}{e^x + e^{-x}} \geq \frac{\sin(x)}{e^x + e^{-x}} \quad \text{for } x \in [0, \pi/4]$$

Therefore,

$$\int_0^{\pi/4} \frac{\cos(x)}{e^x + e^{-x}} dx \geq \int_0^{\pi/4} \frac{\sin(x)}{e^x + e^{-x}} dx$$

2. Find $f'(-10)$ for the following function:

$$f(x) = \int_0^{\ln|x|} e^{-t} dt$$

Let $y = \ln|x|$, $dy = \frac{1}{x} dx$

Then $f'(y) = e^{-y} dy$ by F.T.C. I

And by the chain rule:

$$f'(x) = \frac{e^{-\ln|x|}}{x}$$

$$f'(-10) = \frac{1}{-100}$$

3. Points on the upper half of the unit circle can be written in the form $(\cos(2t), \sin(2t))$ for $0 \leq t \leq \frac{\pi}{2}$.

a.) Write a function of t that gives the distance from $(\cos(2t), \sin(2t))$ to $(1, 0)$.

$$\begin{aligned} d(t) &= \sqrt{(1 - \cos(2t))^2 + \sin^2(2t)} \\ &= \sqrt{2 - 2\cos(2t)} \end{aligned}$$

b.) Find the average distance from points on the upper half of the unit circle to the point $(1, 0)$.
(Hint: use the identity $1 - \cos(2t) = 2\sin^2(t)$ to simplify the function obtained above.)

Average distance is given by:

$$\begin{aligned} \frac{1}{\frac{\pi}{2} - 0} \int_0^{\pi/2} d(t) dt &= \frac{2}{\pi} \int_0^{\pi/2} \sqrt{2 - 2\cos(2t)} dt \\ &= \frac{2}{\pi} \int_0^{\pi/2} 2\sin(t) dt \\ &= \frac{4}{\pi} (-\cos(\pi/2) - (-\cos(0))) \end{aligned}$$

$$= \frac{4}{\pi}$$

4. Evaluate the following definite integrals using the Fundamental Theorem of Calculus:

(a)

$$\int_2^3 \frac{(u+3)(u^2+u-2)}{(u^2+2u-3)} du$$

Since $u^2+u-2 = (u+2)(u-1)$ and $u^2+2u-3 = (u+3)(u-1)$,

we have

$$\int_2^3 \frac{(u+3)(u^2+u-2)}{(u^2+2u-3)} du = \int_2^3 (u+2) du$$

$$= \left(\frac{(3)^2}{2} + 2 \cdot 3 \right) - \left(\frac{(2)^2}{2} + 2 \cdot 2 \right)$$

$$= \frac{9}{2} + 6 - 4 - 4 = \boxed{\frac{9}{2}}$$

(b)

$$\int_{-2}^2 |3x+3| dx$$

Since $|x+1| > 0$ when $x > -1$ and $|x+1| < 0$ when $x < -1$,

$$\int_{-2}^2 |3x+3| dx = 3 \int_{-2}^2 |x+1| dx = 3 \int_{-2}^{-1} (-x-1) dx + 3 \int_{-1}^2 (x+1) dx$$

$$= 3 \left(-\frac{x^2}{2} - x \right) \Big|_{-2}^{-1} + 3 \left(\frac{x^2}{2} + x \right) \Big|_{-1}^2$$

$$= 12 + \frac{3}{2} + \frac{3}{2} - 0$$

$$= \boxed{15}$$