Grade: /10

AMAT112: Calculus I
Worksheet 1

Due: Friday, February 2, in Class or digitally

Name:
UAlbany Email:

—— Instructions ——

- This homework should be submitted in class or digitally on the date listed above.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as Microsoft Lens to take pictures of your work with your phone and convert them into a single pdf file.
- **You must show all work**. You may receive zero or reduced points for insufficient work. **Your work must be neatly organized and written**. You may receive zero or reduced points for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- **Put a box or circle around your final answer** for each question.
- The problems on this assignment will be **graded on correctness and completeness**.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.

1. Let $f(x)$ be a function. In each of the following, we will consider some transformation of f. For each transformation, write an expression for the transformed function and find a function $s(x)$ such that the given transformation of f is equal to either the composition $s \circ f$ or the composition $f \circ s$ (and indicate which one).

   (a) Reflecting across the y-axis.

   (b) Shifting to the right by 2 units followed by stretching horizontally by a factor of 2.

   (c) Vertically shrinking by a factor of $1/3$ followed by reflecting across the x-axis.

2. Let f be a function with

$$\begin{gathered}\text{dom } f = (-\infty, -2) \cup (-2, \infty) \\ \text{range } f = [0, \infty) \\ \text{zeros of } f = \{2, 4\}\end{gathered}$$

Let the function g be defined by

$$g(x) = \frac{\sqrt{f(x-1)}}{f(x+2)}$$

Find the domain, range, and zeros of g.

3. In this problem, we will investigate how symmetry properties are preserved when combining functions. For each combination of the functions f and g, determine the symmetry properties when both functions are even, both are odd, and when f is even and g is odd.

   (a) $f + g$

   (b) $f \cdot g$

(c) $f \circ g$

(d) $g \circ f$

4. A pilot takes off from an airport and flies due east at a speed of 150 km/h for one hour. Then, to avoid bad weather, they turn towards the north, and notify air traffic control that they have turned 45° to the north and are continuing at a speed of 150 km/h. Unfortunately, after one hour they run into bad weather anyway and are forced to make an emergency landing. The landing site is 75 km north and approximately 280 km east of the airport. (*Note:* $280 \approx 75(2+\sqrt{3})$) Investigators determine that one of the plane's navigational instruments gave an incorrect reading. Was the faulty instrument the compass or the speedometer? How far off from the correct reading was it? You may ignore altitude and assume all speeds are relative to the ground.

5. Strontium-90 (^{90}Sr) is a radioactive isotope produced in many nuclear fission reactions. Over time, a sample of ^{90}Sr decays into Yttrium-90 according to the following formula:

$$N(t) = N_0 \cdot 2^{-t/28.8}$$

Where $N(t)$ is the amount of ^{90}Sr in the sample after t years, and N_0 is the initial amount of ^{90}Sr in the sample.

(a) Is the function $N(t)$ one-to-one? Why or why not?

(b) Find a formula for the inverse function of $N(t)$, restricting the domain if necessary.

(c) Give an interpretation of this inverse function. What do the input and output represent?

(d) How many years will it take for 3/4 of the original sample to decay?