
AMAT112: Calculus I

The Mean Value Theorem

1. For each of the following functions, find all numbers c that satisfy the conclusion of the Mean Value Theorem on the indicated interval.

a) $p(x) = 3x^2 - 4x + 7$, $-2 \leq x \leq 5$

b) $v(t) = -3t^3 + 6t^2 - 7t + 1$, $-5 \leq t \leq 1$

c) $g(z) = \frac{4z + 1}{2 - z}$, $-2 \leq z \leq 1$

d) $h(y) = \ln(y^2 + 1)$, $-2 \leq y \leq \frac{1}{2}$

2. Show that each of the following functions has exactly one root.

a) $p(x) = x^5 + x^3 - 7$

b) $f(\theta) = 2x + \cos(x)$

c) $g(t) = t^3 + e^t$

3. Show that, for any real number c , the equation $x^4 + 4x + c = 0$ has at most two real roots.

4. Suppose that $f(5) = 3$ and $4 \leq f'(x) \leq 20$ for $5 \leq x \leq 8$. What is the smallest $f(8)$ can be?

5. Suppose that $f(-18) = 11$ and $3 \leq f'(x) \leq 14$ for $-27 \leq x \leq -18$. What is the largest $f(-27)$ can be?

6. Suppose that $-2 \leq f'(x) \leq 1$ for all x . Show that $-18 \leq f(4) - f(-5) \leq 9$.

7. Does there exist a function f such that $f(1) = 2$, $f(5) = -15$ and $f'(x) \geq -3$ for all x ?