AMAT112: Calculus I

The Fundamental Theorem of Calculus

1. Use the Fundamental Theorem of Calculus to compute the following, without integrating anything.

   a) $\dfrac{d}{dx}\displaystyle\int_0^x \sqrt{1+t^2}\,dt$

   b) $\dfrac{d}{dy}\displaystyle\int_1^y 3x^2\,dx$

   c) $\dfrac{d}{dz}\displaystyle\int_z^5 \sin\left(y^2\right)\,dy$

   d) $\dfrac{d}{dw}\displaystyle\int_w^{-2} \sec\left(z^3\right)\,dz$

   e) $\dfrac{d}{dv}\displaystyle\int_7^{v^2} \ln\left(w^2+1\right)\,dw$

   f) $\dfrac{d}{du}\displaystyle\int_3^{u^3+u} \tan\left(v\right)\,dv$

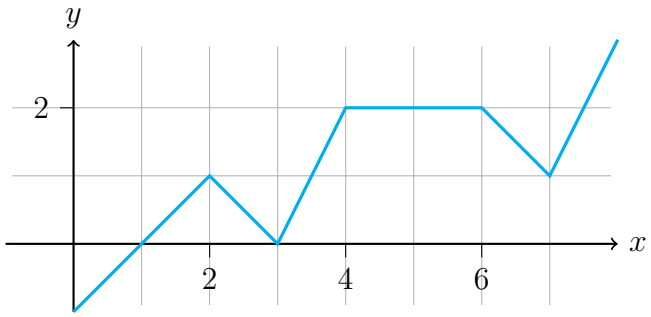
   g) $\dfrac{d}{ds}\displaystyle\int_{\sqrt{s}}^6 \dfrac{u^2}{u^2+4}\,du$

   h) $\dfrac{d}{dr}\displaystyle\int_{\cos(r)}^{\sin(r)} e^{s^2}\,ds$

   i) $\dfrac{d}{dt}\displaystyle\int_{\sqrt{t}}^{t^2} \sin^{-1}\left(r\right)\,dr$

The next three problems are taken from the textbook section 5.4.

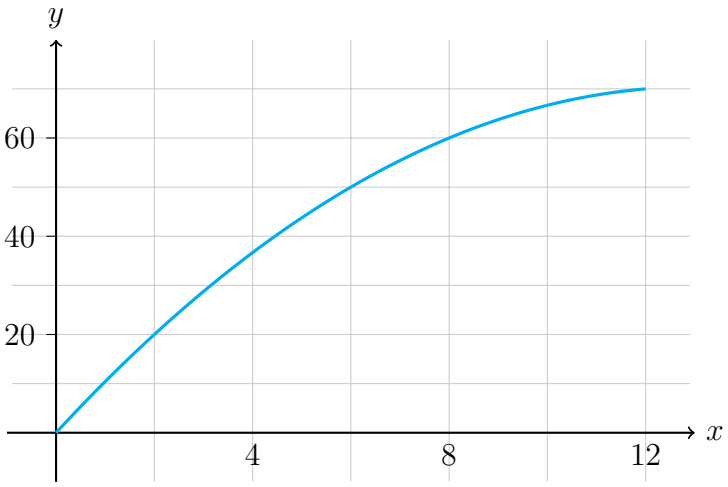
2. Find the average value of f, graphed on the right, on $[0, 8]$.



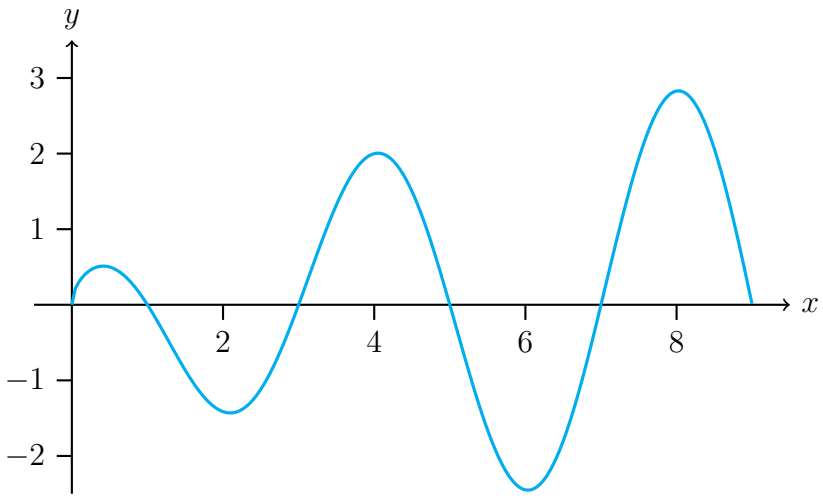
3. The velocity graph of an accelerating car is shown on the right

   a) Estimate the average velocity of the car during the first 12 seconds.

   b) Approximately at what time was the instantaneous velocity equal to the average velocity?



4. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.



a) At what values of x do the local maximum and minimum values of g occur.

b) Where does g attain its absolute maximum value?

c) On what intervals is g concave downward?

d) Sketch the graph of g.

5. The graph of $f(t)$, defined on the interval $[-5, 6.5]$, is given. Define a function by $h(x) = \int_{-5}^x f(t)\,dt$ for $-5 \le x \le 6.5$.

a) Determine the interval(s) where $h(x)$ is increasing.

b) Determine the critical points of $h(x)$.

c) Find all local maximum points.

d) Determine the interval(s) where $h(x)$ is concave down.