AMAT112: Calculus I

The Derivative

1. Use the limit definition of the derivative to calculate the following.

   a) $f(x) = x^3$

   b) $g(t) = \sqrt{t}$

   c) $f(x) = \cos(x)$

   d) $u(s) = \dfrac{1}{s^2}$

   e) $v(t) = (t+1)^2$

   f) $s(a) = \dfrac{1}{a+2}$

2. Use the basic derivative rules to calculate the following.

   a) $\dfrac{d}{dx}\left(x^5 + 5x^4 - 10x^2 + 6\right)$

   b) $\dfrac{d}{dt}\left(3t^{1/2} - t^{3/2} + t^{-1/2}\right)$

   c) $\dfrac{d}{d\theta}\left(\cos(\theta) - 2\theta^2\right)$

   d) $\dfrac{d}{ds}\left(3 \cdot e^s - 4s^2 + 5\right)$

   e) $\dfrac{d}{da}\left(-\sin(a) - \ln(a)\right)$

   f) $\dfrac{d}{dy}\left(\dfrac{1}{2y^2} + \dfrac{4}{\sqrt{y}}\right)$

   g) $\dfrac{d}{dx}\left(\ln(5) + x^e\right)$

   h) $\dfrac{d}{du}\left(\dfrac{2}{\sqrt{u}} + \dfrac{6}{\sqrt[3]{u}}\right)$

   i) $\dfrac{d}{dx}\left(\dfrac{x^3 - 2\sqrt{x}}{x^2}\right)$

   j) $\dfrac{d}{dy}\left(\ln(y) - \ln(y^2)\right)$

   k) $\dfrac{d}{dt}\left(\dfrac{t^3 + t^2 - 4t - 4}{\sqrt{t^3} + 2\sqrt{t}}\right)$

   l) $\dfrac{d}{ds}\left((s+5)^2 (s-5)^2\right)$

   m) $\dfrac{d}{dt}\left(e^t - t^2 + 4t^3\right)$

   n) $\dfrac{d}{dw}\left(\cos(w) + \tan(w)\right)$

   o) $\dfrac{d}{dz}\left(z^4 - \sec(z)\right)$

   p) $\dfrac{d}{dx}\left(\pi x + e\right)$

   q) $\dfrac{d}{dy}\left(\cot(y) - 6y^7\right)$

   r) $\dfrac{d}{dt}\left(15t - 7 + \ln(t)\right)$

   s) $\dfrac{d}{dt}\left(\sqrt{t^7} - \ln(4t)\right)$

   t) $\dfrac{d}{dx}\left(e^x + 9\tan(x)\right)$

   u) $\dfrac{d}{dx}\left(14x^3 + \csc(x) + e^{\pi}\right)$

3. Recall that the derivative of a function at a given point is precisely the slope of the tangent line to that function at that point. With this in mind, find equations for the tangent lines to the given functions at the indicated point.

   a) $f(x) = x + \frac{7}{x}$, $x = 1$

   b) $v(t) = e^t + t$, $t = \ln(2)$

   c) $g(\theta) = \cos(\theta)$, $\theta = \frac{\pi}{4}$

   d) $y(x) = \sqrt{x} + \sqrt[3]{x}$, $x = 64$

   e) $s(a) = (a+1)(a^2 - 3)$, $a = 0$

   f) $\alpha(t) = \sin(t)$, $t = -\frac{\pi}{2}$

4. The *normal line* to a function $f(x)$ at the point $x = a$ is the line perpendicular to the tangent line to $f(x)$ through the point $(a, f(a))$. For the the functions in the previous question, find equations for the normal lines.

5. Recall that the derivative measures how a function is changing. This can mean a multitude of things depending on the setting. The following set of questions are more conceptual than the previous ones, but all come down to understanding what "change" means in each question.

   a) A spherical balloon is being inflated. Find the rate of increase of the surface area ($S = 4\pi r^2$) with respect to the radius, r, when r is 5 cm, 10 cm and 15 cm. What conclusion can you make?

   d) A certain coyote drops an anvil from a height of 31 m. After t seconds the anvils height is $31 - 4.8t^2$ m.

      i. What is the anvil's velocity, speed and acceleration at time t?

      ii. How long will it take the anvil to hit the ground?

      iii. What would be the anvil's velocity at the moment of impact?

      iv. Would Roadrunner have died?

   e) A tennis ball thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s = 24t - 0.8t^2$ m after t seconds.

      i. Find the ball's velocity and acceleration at time t.

      ii. How long does it take the ball to reach its highest point?

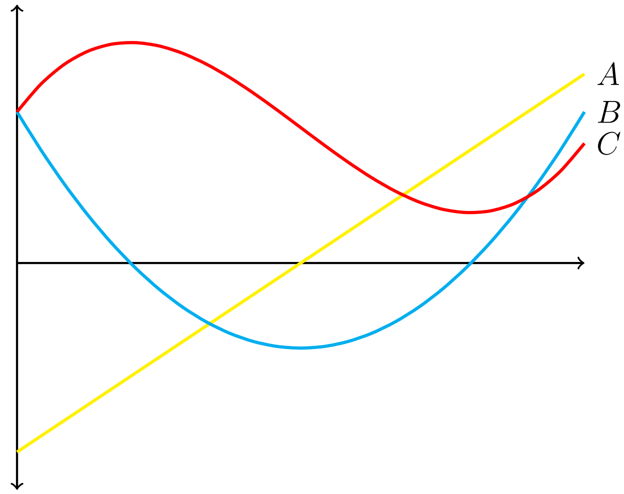
      iii. How high does the ball go?

      iv. How long does it take the ball to reach half of its maximum height?

      v. How long is the ball in the air?

f) The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200\,(30-t)^2$. How fast is the water draining from the tank after 10 minutes? What is the average rate at which the water drains during the first 10 minutes?

g) The graphs in the plot below show the position, s, velocity, v, and acceleration, a, of a body moving along a coordinate line as functions of time t. Determine which graph is which.



6. Let $f(x) = x^2 + x$. In this question we are going to find the equations of both lines through the point $(2, -3)$ that are tangent to the graph of $f(x)$.

a) Find the slope of the line that goes through the points $(a, f(a))$ and $(2, -3)$. (Your answer will be an expression in terms of a).

b) Find the slope of the line tangent to $f(x)$ at the point $(a, f(a))$. (Your answer will be an expression in terms of a).

c) Use your answers in parts a) and b) to find the values of a for which the tangent lines to $f(x)$ at $(a, a^2 + a)$ go through the point $(2, -3)$.

d) For each of the points $(a, f(a))$ found in part c), find the equation of the tangent line to $f(x)$ at $(a, f(a))$.