AMAT112: Calculus I

Optimization

1. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. They need no fence along the river. What are the dimensions of the field that has the largest area?

2. A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

3. Find the point on the parabola $y = 2x$ that is closest to the point $(1, 4)$.

4. A person launches their boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. They could row their boat directly across the river to point C and then run to B, or they could row directly to B, or they could row to some point D between C and B and then run to B. If they can row 6 km/h and run 8 km/h, where should they land to reach B as quickly as possible? (We assume that the speed of the water is negligible compared with the speed at which the person rows.)

5. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

6. We wish to construct a cylindrical container whose volume is $104\,\pi\,\text{cm}^3$. The cost to construct the top of the cylinder is $\$5/\text{cm}^2$, the cost to construct the base of the cylinder is $\$8/\text{cm}^2$ and the cost to construct the body of the cylinder is $\$1/\text{cm}^2$. Determine the dimensions of the cylinder that will minimise the cost of construction.

7. Go to Pauls Online Notes and check out the optimisation problems there.

8. On the next couple of pages are more optimisation problems from a worksheet found on the internet. I unfortunately do not know who the author is to give them credit.

CALCULUS
WORKSHEET ON OPTIMIZATION

Work the following on notebook paper. Write a function for each problem, and justify your answers. Give all decimal answers correct to three decimal places.

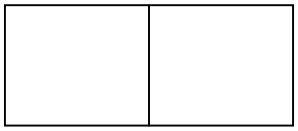
1. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

2. Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.

3. A gardener wants to make a rectangular enclosure using a wall as one side and 120 m of fencing for the other three sides. Express the area in terms of x, and find the value of x that gives the greatest area.

4. A rectangle has a perimeter of 80 cm. If its width is x, express its length and area in terms of x, and find the maximum area.

5. Suppose you had 102 m of fencing to make two side-by-side enclosures as shown. What is the maximum area that you could enclose?



6. Suppose you had to use exactly 200 m of fencing to make either one square enclosure or two separate square enclosures of any size you wished. What plan would give you the least area? What plan would give you the greatest area?

7. A piece of wire 40 cm long is to be cut into two pieces. One piece will be bent to form a circle; the other will be bent to form a square.

(a) Find the lengths of the two pieces that cause the sum of the area of the circle and the area of the square to be a minimum.

(b) How could you make the total area of the circle and the square a maximum?

8. Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?

9. The combined perimeter of an equilateral triangle and a square is 10. Find the dimensions of the triangle and square that produce a minimum total area.

10. The combined perimeter of a circle and a square is 16. Find the dimensions of the circle and square that produce a minimum total area.

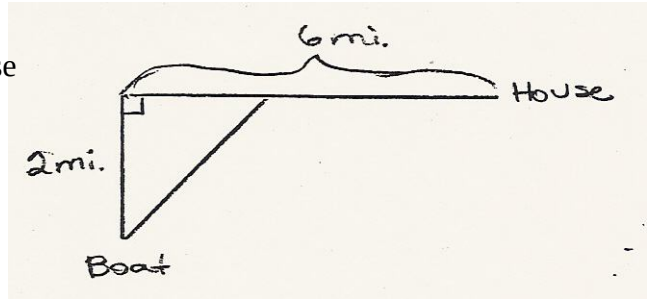
11. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

12. A rectangular page is to contain 24 sq. in. of print. The margins at the top and bottom of the page are each $1\frac{1}{2}$ inches. The margins on each side are 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

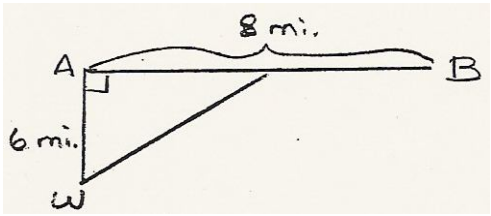
13. A tank with a rectangular base and rectangular sides is open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10/sq. m. for the base and \$5/sq. m. for the sides, what is the cost of the least expensive tank, and what are its dimensions?

14. A cylindrical metal container, open at the top, is to have a capacity of 24π cu. in. The cost of material used for the bottom of the container is \$0.15/sq. in., and the cost of the material used for the curved part is \$0.05/sq. in. Find the dimensions that will minimize the cost of the material, and find the minimum cost.

15. A person in a rowboat two miles from the nearest point on a straight shoreline wishes to reach a house six miles farther down the shore. If the person can row at a rate of 3 mi/h and walk at a rate of 5 mi/h, find the least amount of time required to reach the house. How far from the house should the person land the rowboat?



16. An offshore well is located in the ocean at a point W which is six miles from the closest shore point A on a straight shoreline. The oil is to be piped to a shore point B that is eight miles from A by piping it on a straight line under water from W to some shore point P between A and B and then on to B via a pipe along the shoreline. If the cost of laying pipe is \$100,000 per mile under water and \$75,000 per mile over land, how far from A should the point P be located to minimize the cost of laying the pipe? What will the cost be?



Worksheet on Optimization

1. 24 and 8
2. 50 and 25
3. Area = $x(120-2x)$

   x = 30 ft.
4. Length = $40-x$

   Area = $x(40-x)$

   400 sq. ft.
5. 433.5 sq. m
6. Two squares give 1250 sq. m.
   One square gives 2500 sq. m.
7. (a) Circumference = 17.596 cm and
   perimeter of square = 22.404 cm
   (b) Just a circle with circum.0 of 40 cm
   gives area of 127.324 sq. cm.
8. All 4 ft for the circle; none for the square
9. Sides of triangle = 1.883 m and
   sides of square = 1.087 m
10. Radius of circle = 1.120 and
    sides of square = 2.240
11. 6 in. x 6 in. x 3 in.
12. 9 in x 6 in.
13. $330, 3 x 3 x 4m
14. r = 2in, h = 6in, $5.65
15. 1.733 hr, 4.5 miles
16. $996,862.70, 6.803mi