AMAT112: Calculus I

Linearisation and Differentials

1. Find the linearisation L of the following functions at the indicated points.

   a) $f(x) = \sqrt{x^2+9}, \quad x = -4$

   b) $s(t) = t + \dfrac{1}{t}, \quad t = 1$

   c) $h(s) = \sqrt[3]{s}, \quad s = -8$

   d) $f(\theta) = \tan(\theta), \quad \theta = \pi$

2. Approximate the following quantities using linearisation.

   a) $\sqrt{8.9}$

   b) $\ln(1.1)$

   c) $\sec(0.1)$

3. Find the differential of each of the following.

   a) $y = x\sqrt{1-x^2}$

   b) $s = \dfrac{2\sqrt{t}}{3\left(1+\sqrt{t}\right)}$

   c) $a = \cos(z^2)$

   d) $g = \sec(x^2-1)$

   e) $r = 2\cot\left(\dfrac{1}{\sqrt{t}}\right)$

   f) $y = \ln\left(\dfrac{x+1}{\sqrt{x-1}}\right)$

4. The radius of a sphere was measured and found to be 20 cm with a possible error in measurement of at most 0.01 cm. Find the maximum error and percentage error in using this value of the radius to compute the volume of the sphere.

5. The diameter of a sphere is measured as 100 cm with a possible error in measurement of at most 1 cm. Estimate the maximum error and percentage error in using this measurement in calculating the volume of the sphere.

6. The radius of a circular disk is given as 32 cm with a maximum error in measurement of 0.3 cm. Use differentials to estimate the maximum possible error and percentage error in computing the surface area of the disk.

7. The edge of a cube was found to be 25 cm with a possible error in measurement of 0.05 cm. Use differentials to estimate the maximum possible error and percentage error in computing

   a) the volume of the cube,

   b) the surface area of the cube.

8. The circumference of a sphere was measured to be 81 cm with a possible error of 0.4 cm. Use differentials to estimate the maximum possible error and percentage error in computing

   a) the volume of the sphere,

   b) the surface area of the sphere.

9. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.04 cm thick to a hemispherical dome with diameter 45 m.