AMAT112: Calculus I

L'Hôpital's Rule

1. For each of the following limits determine which indeterminate form the expression corresponds to, then calculate the limit using L'Hôpital's Rule.

a) $\lim\limits_{x\to\infty} \dfrac{2x^2+3x}{x^3+x+1}$

b) $\lim\limits_{y\to -1} \dfrac{3y^3+3}{4y^3-y+3}$

c) $\lim\limits_{\theta\to 0} \dfrac{\sin(5\theta)}{2\theta}$

d) $\lim\limits_{t\to 0} \dfrac{\sin(t)-t}{t^3}$

e) $\lim\limits_{z\to e^+} \ln(z)^{1/(z-e)}$

f) $\lim\limits_{\theta\to -\pi/3} \dfrac{3\theta+\pi}{\sin(\theta+\pi/3)}$

g) $\lim\limits_{t\to 1} \dfrac{t-1}{\ln(t)-\sin(\pi t)}$

h) $\lim\limits_{\theta\to \pi/2} \dfrac{\ln(\csc(\theta))}{(\theta-\pi/2)^2}$

i) $\lim\limits_{x\to 0^+} \left(1+\dfrac{1}{x}\right)^x$

j) $\lim\limits_{t\to 0} \dfrac{t\sin(t)}{1-\cos(t)}$

k) $\lim\limits_{\theta\to(\pi/2)^-} \left(\dfrac{\pi}{2}-\theta\right)\tan(\theta)$

l) $\lim\limits_{u\to 0} \dfrac{\left(\frac{1}{2}\right)^u-1}{u}$

m) $\lim\limits_{w\to 0} \dfrac{3^w-1}{2^w-1}$

n) $\lim\limits_{x\to\infty} \dfrac{\log_2(x)}{\log_3(x+3)}$

o) $\lim\limits_{s\to 0^+} \dfrac{\ln(e^s-1)}{\ln(s)}$

p) $\lim\limits_{x\to\infty} \left(\dfrac{x^2+1}{x+2}\right)^{1/x}$

q) $\lim\limits_{\theta\to 0^+} \ln(\theta)-\ln(\sin(\theta))$

r) $\lim\limits_{t\to 0^+} \dfrac{3t+1}{t}-\dfrac{1}{\sin(t)}$

s) $\lim\limits_{\theta\to 0^+} \csc(\theta)-\cot(\theta)+\cos(\theta)$

t) $\lim\limits_{y\to 0} \dfrac{e^y-(1+y)}{y^2}$

u) $\lim\limits_{x\to\infty} x^2e^{-x}$

v) $\lim\limits_{x\to\infty} x^{1/\ln(x)}$

w) $\lim\limits_{t\to 0} \dfrac{(e^t-1)^2}{t\sin(t)}$

x) $\lim\limits_{\theta\to 0} \dfrac{\sin(3\theta)-3\theta+\theta^2}{\sin(\theta)\sin(2\theta)}$

y) $\lim\limits_{x\to 1^+} x^{1/(x-1)}$

z) $\lim\limits_{y\to 0} (e^y+y)^{1/y}$

2. Calculate the following limits.

a) $\lim_{t\to 0} \dfrac{\sin(5t)}{t}$

b) $\lim_{x\to\infty} \dfrac{\sqrt{9x+1}}{\sqrt{x+1}}$

c) $\lim_{v\to 0} \dfrac{5^v - 4^v}{3^v - 2^v}$

d) $\lim_{x\to\infty} x - \ln(x)$

e) $\lim_{z\to 0^+} z\ln(z)^2$

f) $\lim_{\theta\to 0^+} \dfrac{\sqrt{\theta}}{\sqrt{\sin(\theta)}}$

g) $\lim_{t\to 0} \dfrac{t}{\arctan(4t)}$

h) $\lim_{x\to 1} \dfrac{x^3 - 1}{4x^3 - x - 3}$

i) $\lim_{t\to 0} \dfrac{\arcsin(t)}{\sin(t)}$

j) $\lim_{y\to 0} \dfrac{y5^y}{5^y - 1}$

k) $\lim_{\theta\to(\pi/2)^-} \dfrac{\sec(\theta)}{\tan(\theta)}$

l) $\lim_{x\to\infty} x - \sqrt{x^2 + x}$

m) $\lim_{s\to 3} \dfrac{s-3}{s^2 - 3}$

n) $\lim_{\theta\to\infty} \theta\tan\left(\dfrac{1}{\theta}\right)$

o) $\lim_{x\to\infty} \dfrac{2^x - 3^x}{3^x + 4^x}$

p) $\lim_{h\to 0^+} \dfrac{1}{h} - \dfrac{1}{e^h - 1}$

q) $\lim_{t\to 0^+} \sin(t)\ln(t)$

r) $\lim_{\theta\to(\pi/2)^-} e^{-\tan(\theta)}\sec^2(\theta)$

s) $\lim_{x\to\infty} \dfrac{e^{x^2}}{xe^x}$

t) $\lim_{t\to 0^+} \ln(t)\tan\left(\dfrac{\pi x}{2}\right)$

u) $\lim_{x\to\infty} \left(1 + \dfrac{\pi}{x}\right)^x$

v) $\lim_{\theta\to(\pi/2)^-} \tan(\theta)^{\cos(\theta)}$

w) $\lim_{z\to 0^+} \dfrac{z}{e^{-1/z}}$

x) $\lim_{x\to 0^+} x^x$

y) $\lim_{t\to 1} t^{\tan(\pi t/2)}$

z) $\lim_{x\to-\infty} \dfrac{2^x + 4^x}{5^x - 2^x}$