AMAT112: Calculus I

Inverse Functions

1. Find a formula for the inverse of each of the following functions.

   a) $f(x) = 2x + 3$

   b) $h(z) = \dfrac{5 - 2z}{3z + 7}$

   c) $s(t) = -\dfrac{1}{3}t^2 + 4t - 5, \quad t \geq 6$

   d) $g(x) = x^3 - 6x^2 + 12x - 8$

2. For each of the following, calculate $\dfrac{d}{dx}\left(f^{-1}(x)\right)\Big|_{x=a}$.

   a) $f(x) = \dfrac{3x + 1}{4x + 1}, \quad a = -1$

   b) $f(x) = x^3 - x^2 + 2x + 2, \quad a = 4$

   c) $f(x) = 3x^3 + 3\sin(x) + 4\cos(x), \quad a = 4$

   d) $f(x) = \sin(x) + \cos(x), \quad a = \sqrt{2}$

3. Given the table below, find the following quantities.

x	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	-1	2	-2	-4	1	0	4	5	-3	3
$f'(x)$	9	-6	-1	6	7	0	5	0	-4	-7
$g(x)$	-2	3	5	4	-3	-1	1	2	-4	0
$g'(x)$	5	6	-4	3	8	9	2	3	5	6
$h(x)$	3	2	-4	2	-3	5	-1	0	4	-2
$h'(x)$	9	4	0	2	-3	9	3	-4	-5	2

   a) $(f \circ g)'(5)$ b) $(h \cdot f)'(3)$ c) $\left(\dfrac{h}{g}\right)'(1)$ d) $(f^{-1})'(-4)$ e) $(g^{-1})'(-1)$

   f) $(f^{-1} \circ h)'(-3)$ g) $(h \cdot g^{-1})'(4)$ h) $\left(\dfrac{f^{-1}}{g^{-1}}\right)'(0)$ i) $(g \circ h^{-1})'(2)$ j) $(f \cdot g^{-1})'(-2)$

4. Find the exact value of each of the following.

a) $\arctan(1)$

b) $\arcsin\left(-\frac{1}{2}\right)$

c) $\arccos\left(\frac{\sqrt{3}}{2}\right)$

d) $\operatorname{arccsc}\left(\sqrt{2}\right)$

e) $\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right)$

f) $\operatorname{arccot}\left(\sqrt{3}\right)$

g) $\sin\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$

h) $\tan\left(\arcsin\left(-\frac{1}{2}\right)\right)$

i) $\sec\left(\arccos\left(\frac{1}{2}\right)\right)$

j) $\cot\left(\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right)$

k) $\tan(\arctan(10))$

l) $\arcsin\left(\sin\left(\frac{7\pi}{3}\right)\right)$

m) $\cos\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)$

n) $\arctan\left(\tan\left(\frac{5\pi}{6}\right)\right)$

o) $\operatorname{arccot}\left(\cot\left(-\frac{\pi}{6}\right)\right)$

5. Fill in the table below with expressions for $f(g(x))$ that are completely free of trigonometric functions.

$f(x)$ \ $g(x)$	$\arcsin(x)$	$\arccos(x)$	$\arctan(x)$	$\operatorname{arccot}(x)$	$\operatorname{arcsec}(x)^*$	$\operatorname{arccsc}(x)^*$
$\sin(x)$	x					
$\cos(x)$		x				
$\tan(x)$			x			
$\cot(x)$				x		
$\sec(x)$					x	
$\csc(x)$						x

* *You will not be tested on compositions where* $g(x) = \operatorname{arcsec}(x)$ *or* $g(x) = \operatorname{arccsc}(x)$.

6. Simplify each of the following expressions.

a) $\sin\left(2\operatorname{arccot}\left(x\right)\right)$

b) $\sin\left(\arccos(x)+\arctan\left(x\right)\right)$

c) $\cos\left(2\operatorname{arccot}\left(x\right)+\arcsin\left(x\right)\right)$

d) $\tan\left(\arccos\left(\sqrt{x^2-1}\right)\right)$

e) $\sin\left(\arctan\left(\dfrac{x^2}{2}\right)\right)$

f) $\cos\left(2\operatorname{arccot}\left(\dfrac{x}{4}\right)+\arctan\left(\dfrac{1}{x}\right)\right)$

7. Find the derivative of each of the following.

a) $f(x)=\ln\left(\arctan\left(x\right)\right)$

b) $h(z)=\operatorname{arccot}\left(\sqrt{z}\right)$

c) $g(y)=\operatorname{arccot}\left(\sqrt{y-1}\right)$

d) $u(t)=\arccos\left(e^{-t}\right)$

e) $W\left(\theta\right)=\operatorname{arccot}\left(\cos\left(\theta\right)\right)$

f) $w(s)=\sqrt{s^2-1}-\operatorname{arcsec}\left(s\right)$

g) $f(z)=\operatorname{arccot}\left(\dfrac{1}{z}\right)-\arctan\left(z\right)$

h) $F(x)=\arctan\left(\sqrt{x^2-1}\right)+\operatorname{arccsc}\left(x\right)$

i) $s\left(\theta\right)=\theta\arcsin\left(\theta\right)+\sqrt{1-\theta^2}$

j) $h(x)=\ln\left(x^2+4\right)-x\arctan\left(\dfrac{x}{2}\right)$

k) $y\left(t\right)=\arcsin\left(\sqrt{\sin\left(t\right)}\right)$

l) $G(x)=\arctan\left(\sqrt{\dfrac{1-x}{1+x}}\right)$