AMAT112: Calculus I

Implicit Differentiation

1. Use implicit differentiation to find $\dfrac{dy}{dx}$ for each of the following curves.

   a) $x^2y + xy^2 = 6$

   b) $x^3 + y^3 = 12xy$

   c) $x^2\left(x-y\right)^2 = x^2 - y^2$

   d) $\left(3xy+7\right)^2 = 6y$

   e) $x^y = y^x$

   f) $x^3 = \dfrac{2x-y}{x+3y}$

   g) $xy = \cot\left(xy\right)$

   h) $x^4 + \sin\left(y\right) = x^3y^2$

   i) $y = \ln\left(x^2+y^2\right)$

   j) $y\sin\left(\dfrac{1}{y}\right) = 1 - xy$

   k) $x\cos\left(2x+3y\right) = y\sin\left(x\right)$

   l) $e^{2x} = \sin\left(x+3y\right)$

   m) $e^{x^2y} = 2x + 2y$

   n) $e^{\tan(y)} = \ln\left(xy^2 + x^3y\right)$

2. Find the equation of the tangent lines to the following curves at the given point.

   a) $x^2y^2 = 9, \quad (-1,3)$

   b) $y^2 - 2x - 4y - 1 = 0, \quad (-2,1)$

   c) $x^2 - xy - y^2 = 1, \quad (2,1)$

   d) $6x^2 + 3xy + 2y^2 + 17y - 6 = 0, \quad (-1,0)$

   e) $x^2 - \sqrt{3}xy + 2y^2 = 5, \quad \left(\sqrt{3},2\right)$

   f) $2xy + \pi\sin\left(y\right) = 2\pi, \quad \left(1,\dfrac{\pi}{2}\right)$

   g) $y\sin\left(2x\right) = x\cos\left(2y\right), \quad \left(\dfrac{\pi}{2},\dfrac{\pi}{4}\right)$

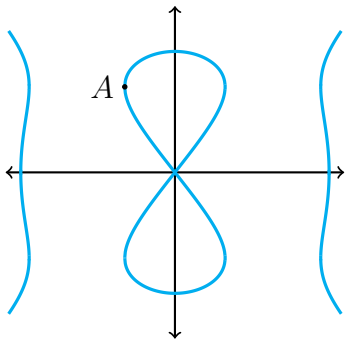
   h) $x\sin\left(2y\right) = y\cos\left(2x\right), \quad \left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$

   i) $y = 2\sin\left(\pi x - y\right), \quad (1,0)$

   j) $x^2\cos^2\left(y\right) - \sin\left(y\right) = 0, \quad (0,\pi)$

3. Consider the curve given below.

$$8y^4 - 8y^2 - 8x^4 + 13x^2 = 0$$



a) Find an expression for $\frac{dy}{dx}$.

b) Find all points on the curve where $x = \frac{\sqrt{2}}{4}$.

c) Find the slope of a tangent line to the curve at $x = \frac{\sqrt{2}}{4}$. You may use any of the points found in (b).

d) Find the coordinates of point A.

4. Show that the sum of the x- and y- intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c.

5. Consider the curve $y^4 - 4y^2 - x^4 + 9x^2 = 0$.

a) Find an expression for $\frac{dy}{dx}$.

b) Find the equations of each of the tangent lines to the curve at all points where $x = 3$.

Answers

1.

a) $\dfrac{dy}{dx} = -\dfrac{y\,(y+2x)}{x\,(x+2y)}$

b) $\dfrac{dy}{dx} = -\dfrac{x^2-4y}{y^2-4x}$

c) $\dfrac{dy}{dx} = \dfrac{x\,(2x^2-3xy+y^2-1)}{x^3-x^2y-y}$

d) $\dfrac{dy}{dx} = \dfrac{y\,(3xy+7)}{3x^2y+7x-1}$

e) $\dfrac{dy}{dx} = \dfrac{y}{y\,(y^{x+1}-x^{y+1}\ln(y))}x\,(x^{y+1}-y^{x+1}\ln(x))$

f) $\dfrac{dy}{dx} = -\dfrac{3x^2(x+3y)^2-7y}{7x}$

g) $\dfrac{dy}{dx} = -\dfrac{y}{x}$

h) $\dfrac{dy}{dx} = \dfrac{x^2(3y^2-4x)}{\cos(y)-2x^3y}$

i) $\dfrac{dy}{dx} = \dfrac{2x}{x^2+y^2}$

j) $\dfrac{dy}{dx} = -\dfrac{y^2}{xy+y\sin(1/y)-\cos(1/y)}$

k) $\dfrac{dy}{dx} = \dfrac{\cos(2x+3y)-2x\sin(2x+3y)-y\cos(x)}{\sin(x)+3x\sin(2x+3y)}$

l) $\dfrac{dy}{dx} = \dfrac{1}{3}\,(2e^{2x}\sec(x+3y)-1)$

m) $\dfrac{dy}{dx} = -\dfrac{2\left(xye^{x^2y}-1\right)}{x^2e^{x^2y}-2}$

n) $\dfrac{dy}{dx} = \dfrac{y^2+3x^2y}{(xy^2+x^3y)\sec^2(y)\,e^{\tan(y)}-2xy+x^3}$

2.

a) $y-3=3\,(x+1)$

b) $y-1=-\,(x+2)$

c) $y-1=\dfrac{3}{4}\,(x-2)$

d) $y=\dfrac{6}{7}\,(x+1)$

e) $y-2=0$

f) $y-\dfrac{\pi}{2}=-\dfrac{\pi}{2}\,(x-1)$

g) $y-\dfrac{\pi}{4}=\dfrac{1}{2}\left(x-\dfrac{\pi}{2}\right)$

h) $y-\dfrac{\pi}{2}=2\left(x-\dfrac{\pi}{4}\right)$

i) $y=\dfrac{2\pi}{2\pi-1}\,(x-1)$

j) $y-\pi=0$