AMAT112: Calculus I

Evaluating Definite Integrals

1. Compute the following integrals.

a) $\displaystyle\int_{-1}^{2} 3x^2 - 2x + 1\,dx$

b) $\displaystyle\int_{-1}^{0} x - x^2\,dx$

c) $\displaystyle\int 2\sec^2(\theta)\,d\theta$

d) $\displaystyle\int_{0}^{1} 3x^2 + x - 5\,dx$

e) $\displaystyle\int (y+1)^2\,dy$

f) $\displaystyle\int_{0}^{1/2} \frac{4}{\sqrt{1-x^2}}\,dx$

g) $\displaystyle\int_{-\pi/3}^{-\pi/4} 4\sec^2(\theta) + \frac{\pi}{\theta^2}\,d\theta$

h) $\displaystyle\int \frac{z^5 - 2z}{z^3}$

i) $\displaystyle\int \frac{3}{1+v^2} - \csc^2(v)\,dv$

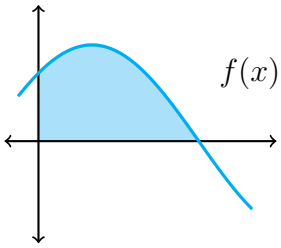
j) $\displaystyle\int_{1}^{\sqrt{2}} \frac{t^2 + \sqrt{t}}{t^2}\,dt$

k) $\displaystyle\int \frac{\left(x^{1/3}+1\right)\left(2 - x^{2/3}\right)}{x^{1/3}}\,dx$

l) $\displaystyle\int_{0}^{\pi/3} (\sec(\theta) + \tan(\theta))^2\,d\theta$

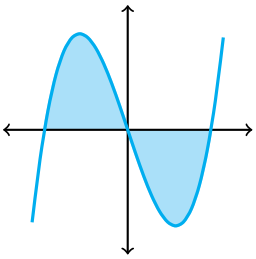
2. Find the area of the indicated shaded regions.

a)



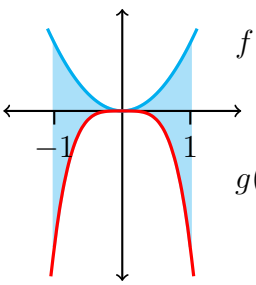
$f(x) = \cos(x) + \sin(x)$

b)



$f(x) = 3(x^3 - x)$

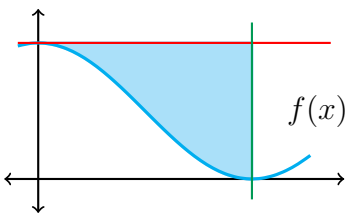
c)



$f(x) = x^2$

$g(x) = -2x^4$

d)



$f(x) = 1 + \cos(x)$

3. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)\,dt$ represent?

4. If $f(x)$ is the slope of a trail a distance of x miles from the start of the trail, what does $\int_3^5 f(x)\,dx$ represent?

5. The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find the

   (i) velocity at time t

   (ii) distance travelled during the given time interval.

   a) $a(t) = t + 4$, $v(0) = 5$, $0 \leq t \leq 10$

   b) $a(t) = 2t + 3$, $v(0) = -4$, $0 \leq t \leq 3$

6. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ litres per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.