AMAT112: Calculus I

Derivatives and Curve Sketching

You might want to use Desmos to check your answers for this worksheet. For every sketch you create, you should clearly label intercepts, asymptotes, extrema and inflection points.

1. Determine the intervals where the following functions are concave up and concave down. Also find all points of inflection.

   a) $f(x) = x^3 - 3x + 3$ b) $f(x) = -2x^3 + 6x^2 - 3$ c) $f(x) = x^4 - 2x^2$

   d) $f(x) = 4x^3 - x^4$ e) $f(x) = x + \sin(x)$ f) $f(x) = \frac{4}{3}x - \tan(x)$

   g) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$ h) $f(x) = x\sqrt{8-x^2}$ i) $f(x) = \dfrac{8x}{x^2+4}$

   j) $f(x) = xe^{1/x}$ k) $f(x) = \dfrac{\ln(x)}{\sqrt{x}}$ l) $f(x) = \dfrac{e^x}{1+e^x}$

   Paul's Online Notes has many more questions like this that you might want to check out. Both sections 4.5 and 4,6 are worth looking at.

2. Sketch the graphs of the following functions.

   a) $f(x) = (x-1)^3(x+4)$, $f'(x) = (x-1)^2(4x+11)$, $f''(x) = 6(x-1)(2x+3)$

   b) $f(x) = \dfrac{e^x}{x}$, $f'(x) = \dfrac{e^x(x-1)}{x^2}$, $f''(x) = \dfrac{e^x(x^2-2x+2)}{x^3}$

   c) $f(x) = \dfrac{x-3}{\sqrt{x^2+1}}$, $f'(x) = \dfrac{3x+1}{(x^2+1)^{3/2}}$, $f''(x) = -\dfrac{3(x+1)(2x-1)}{(x^2+1)^{5/2}}$

   d) $f(x) = \dfrac{x^2}{\sqrt{e^x}}$, $f'(x) = -\dfrac{x(x-4)}{2\sqrt{x}}$, $f''(x) = \dfrac{x^2-8x+8}{4\sqrt{e^x}}$

3. Sketch the following curves.

   a) $y = \dfrac{2x^2+x-1}{x^2-1}$ b) $y = \dfrac{x^3-3x^2+3x-1}{x^2+x-2}$ c) $y = \dfrac{8}{x^2+4}$

   d) $y = \dfrac{4x}{x^2+4}$ e) $y = \sqrt{x}(x-1)$ f) $y = \sqrt{x}e^{-x^2}$