
AMAT112: Calculus I

Chain, Product and Quotient Rules

Chain Rule: $\frac{d}{dx} (f(g(x))) = f'(g(x)) \cdot g'(x)$

Product Rule: $\frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

1. Use the chain rule to find the derivatives of the following functions.

a) $y = \sqrt{5-x}$

b) $g = \sqrt[3]{2t-t^2}$

c) $h(y) = (1+2y)^4$

d) $s(t) = (3-2t^2)^{-5}$

e) $f(x) = e^{\cos(x)}$

f) $u(z) = \sin(3z^2 - 2z + 1)$

g) $y = \ln(\sin(x))$

h) $y = \tan^3(\theta)$

i) $g(z) = \sqrt{\ln(x)}$

j) $f(x) = \sin(5x) - \cos(2x)$

k) $A = (\csc(s) + \cot(s))^{-1}$

l) $a(t) = (\sin(t) + \cos(t))^5$

m) $h(x) = \sin^2(3x)$

n) $p(s) = \ln(\cos(s))^2$

o) $y = \sin^3(3t+4)$

p) $f(\theta) = \sqrt{\sin(\sqrt{\theta})}$

2. Use the product rule (and maybe the chain rule) to find the derivatives of the following functions.

a) $y = x \sin(x)$

b) $g = te^t$

c) $h(y) = e^y \cos(y)$

d) $s(t) = 3t \ln(t)$

e) $f(x) = \sin(x) \ln(x)$

f) $u(z) = e^z (z^2 + 1)$

g) $y = (3 - 4x^2)x^{-1/2}$

h) $y = (1 + \sin(\theta))(1 - \sin(\theta))$

i) $g(z) = \sqrt{z} \ln(z)$

j) $f(x) = e^{2x+1}(2x + 1)$

k) $A = \cos(5s) \sin(3s)$

l) $a(t) = t^2 \ln(t^2)$

m) $h(x) = (x^2 + 3x)(e^x - 1)(\sqrt{x} - \sqrt[3]{x})$

n) $p(s) = (e^{3s} + 5) \ln(\cos(s))^2$

o) $y = (t + 1)(\sqrt{t^2 - 1})^{-1}$

p) $f(\theta) = \csc(\theta) \tan(\theta) \cot(\theta) \sin(\theta)$

3. Use the quotient rule to find the derivatives of the following functions.

a) $y = \frac{x+1}{x-1}$

b) $g = \frac{\sin(t)}{t}$

c) $h(y) = \frac{\sin(y)}{e^y}$

d) $s(t) = \frac{t^2 + 1}{t + 1}$

e) $f(x) = \frac{\cos(x)}{\sqrt{x}}$

f) $u(z) = \frac{\ln(z)}{z}$

g) $y = \frac{x}{\cos(x)}$

h) $y = \frac{\theta + \sin(\theta)}{\cos(\theta)}$

i) $g(z) = \frac{z^2}{1 - \sqrt{z}}$

j) $f(x) = \frac{x^2 - 3x + 1}{\sqrt{x}}$

4. Find the derivatives of the following functions.

a) $y = \frac{x^3}{(x+3)^5}$

b) $g = (t+1)^6 \sqrt{t-1}$

c) $h(y) = \frac{2y-1}{\sqrt{y+1}}$

d) $s(t) = \frac{(t+2)^2 (3t-4t^5)^{420}}{(8-t)^1 8}$

e) $f(x) = \left(\frac{x^2}{x^3-x} \right)^3$

f) $u(z) = \left(\frac{3z-4}{5z+2} \right)^{-5}$

g) $y = \cos \left(5 \sin \left(\frac{x}{3} \right) \right)$

h) $y = 4 \sin \left(\sqrt{1+\sqrt{\theta}} \right)$

i) $g(z) = \sqrt{3z + \sqrt{2 + \sqrt{1-z}}}$

j) $f(x) = \frac{1 + \cos^2(7x)}{\sqrt{x^3+1}}$

5. If you really want to practice your algebra, find the second derivative of all the problems on this worksheet.