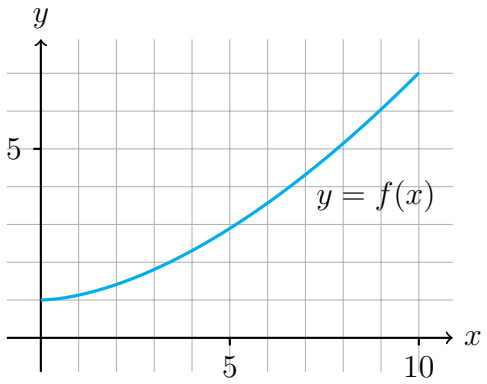
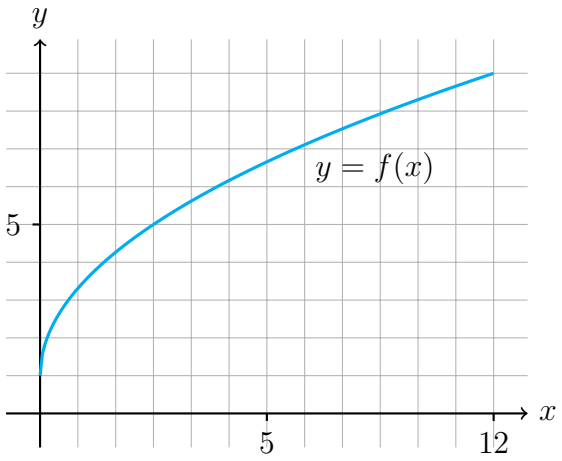
AMAT112: Calculus I

Area and the Definite Integral

1. By reading values from the given graph of f, use five rectangles to find a lower and an upper estimate for the area under the given graph of f from $x = 0$ to $x = 10$.



2. a) Use six rectangles to find estimates of each type for the area under the graph of f from $x = 0$ to $x = 12$.
   (i) L_6 (left endpoints)
   (i) R_6 (right endpoints)
   (i) M_6 (midpoints)



   b) Is L_6 an underestimate or overestimate of the true area?

   c) Is R_6 an underestimate or overestimate of the true area?

   d) Which of the numbers L_6, R_6 or M_6 gives the best estimate of the true area? Explain.

3. You and a companion are about the drive a twisty stretch of dirt road in a car whose speedometer works but whose odometer is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-second intervals, with the results shown in the table below.

Time (s)	0	10	20	30	40	50	60	70	80	90	100	110	120
Velocity (ft/s)	0	44	15	35	30	44	35	15	22	35	44	30	35

   Estimate the length of the road using

   a) left-endpoints.

   b) right-endpoints.

4. You are sitting on the bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for an hour, with the results shown in the table below.

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Velocity (m/s)	1	1.2	1.7	2.0	1.8	1.6	1.4	1.2	1.0	1.8	1.5	1.2	0

About how far upstream did the bottle travel during that hour? Find an estimate using

a) left-endpoints.

b) right-endpoints.

5. For each of the following functions, evaluate the Riemann sum using 6 subintervals and each type of sample point.

(i) L_6 (left endpoints) (ii) R_6 (right endpoints) (iii) M_6 (midpoints)

a) $y = x^3$, between $x = 0$ and $x = 1$.

b) $y = \dfrac{1}{x}$, between $x = 1$ and $x = 7$.

c) $y = 4 - x^2$, between $x = -3$ and $x = 3$

d) $y = \sin(x)$, between $x = 0$ and $x = \pi$,

6. Suppose

$$\int_1^2 f(x)\,dx = -4, \quad \int_1^5 f(x)\,dx = 3, \quad \int_1^5 g(x) = 6.$$

Compute the following.

a) $\displaystyle\int_2^2 g(x)\,dx$

b) $\displaystyle\int_5^1 g(x)\,dx$

c) $\displaystyle\int_1^2 3f(x)\,dx$

d) $\displaystyle\int_2^5 f(x)\,dx$

e) $\displaystyle\int_1^5 f(x) - g(x)\,dx$

f) $\displaystyle\int_1^5 4f(x) - g(x)\,dx$

7. Evaluate the following integrals by interpreting them as areas.

a) $\displaystyle\int_{-1}^2 1 - x\,dx$

b) $\displaystyle\int_0^9 \frac{1}{3}x - 2\,dx$

c) $\displaystyle\int_{-3}^0 1 + \sqrt{9 - x^2}\,dx$

d) $\displaystyle\int_{-5}^5 x - \sqrt{25 - x^2}\,dx$

e) $\displaystyle\int_{-1}^2 |x|\,dx$

f) $\displaystyle\int_0^{10} |x - 5|\,dx$